

INTERNATIONAL CONFERENCE ON HOLOGRAPHY AND ITS APPLICATIONS

ICHA 2026

Damghan University

PROCEEDINGS

The 5th International Conference on Holography and its Applications

18–19 Shahrivar 1405 • 9–10 September 2026

Damghan University

PREPARED BY

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Damghan University

International Conference on Holography and its Application

9 and 10 September 2026

Damghan University, Damghan, Iran.

Important Dates:

**Abstract & Paper Submission Deadline:
July 22, 2026**

**Registration Deadline:
August 11, 2026**



Conference Topics:

1. Foundations of the Holographic Principle and AdS/CFT Correspondence
2. Holographic aspects of Quantum Information and Quantum Computing
3. Holography and Strongly Correlated Systems
4. Holography and Cosmology
5. Machine Learning for the Holographic Principle

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The 5th International Conference on Holography and its Applications 2026

Conference Schedule

Timezone Notice: All scheduled presentation times are listed in Iran Standard Time (IRST).

Day / Date	Time (IRST)	Speaker	Presentation Title & Topic Details	Chairman
Wednesday Sept 9th	8:30 - 9:00 AM	Opening		B. Pourhassan
	09:00 - 10:00 AM	Yang-Hui He	Calabi-Yau manifolds: from geometry, to physics, to AI-driven Discovery	
	10:00 - 10:30 AM	Mehdi Sadeghi	Modified Yang-Mills AdS black brane and DC conductivity	
	10:30 - 11:00 AM	Jonas Mager	Hadronic light-by-light scattering and the muon g-2 from AdS/QCD	
	11 - 11:30 AM	Ebrahim Yusofi	Emergent Anti-Gravity from Entropic Forces on Gpc-Scale: A Holographic Origin of Dynamical Dark Energy	
	11:30 AM - 02:00 PM		No Sessions	M.R. Mohammadi-Mozaffar
	02:00 - 03:00 PM	Mohsen Alishahiha	Beyond Krylov Complexity	
	03:00 - 03:30 PM	Moisés Bravo	Rotation and Anisotropic Scaling in Axionic Holographic Superconductors: AdS4 versus Lifshitz5	
	03:30 - 04:00 PM	Sedigheh Farahzad	Effects of boundary variables in the Holographic dark energy model	
	04:00 - 04:15 PM Poster Session	Fatemeh Khosravani Amiri — Phenomenological Consequences of AdS-Scale Deformations in RPS Holography of Charged AdS Black Holes Bahman Khanpour — The entropic force and real time dynamics of heavy quarkonia in Yang-Mills-dilaton black holes		

Timezone Notice: All scheduled presentation times are listed in Iran Standard Time (IRST).

Day / Date	Time (IRST)	Speaker	Presentation Title & Topic Details	Chairman
Thursday Sept 10th	09:00 - 10:00 AM	Mukund Rangamani	Imprints of the black hole singularity in field theory observables	M.R. Garousi
	10:00 - 11:00 AM	Shahin Sheikh-Jabbari	GR from RG	
	11:00 - 11:30 AM	Sayyed Mehrab Ramezani	Thermodynamics and Phase Structure of Charged Simpson-Visser-AdS Black Holes	
	11:30 AM - 12:00 PM	Zaharia Adrian	Thermal string field theory, Hagedorn scaling, and de Sitter observer algebras	
	12:00 - 02:00 PM		No Sessions	H. R. Afshar
	02:00 - 02:30 PM	Narmin Nasibova	Pion Charge Density in the Thermal Soft-Wall Model of Holographic QCD	
	02:30 - 03:00 PM	Xun Chen	Machine learning in holographic QCD phase transition	
	03:00 - 03:30 PM	Hiwa Abubakr Ahmed	Probing Chiral and U(1) Axial Symmetry Restoration via Meson Susceptibilities in Holographic QCD	
	03:30 - 04:00 PM	Vahid Taghiloo	Dynamical Gravitational Entropy Is a Noether Charge	
	04:00 - 05:00 PM	Hong Liu	Euclidean Wormholes, Quantum Chaos, and the Factorization Puzzle	

Abstract - Presentation

Keynote Speaker

Imprints of the black hole singularity in field theory observables

Mukund Rangamani¹.

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Abstract. I will review recent progress on understanding how the black hole singularity is encoded in thermal observables. Specifically, the non-perturbative high frequency part of the thermal spectral function carries the relevant information. For holographic CFTs, where the correlators can be computed using bulk semiclassical physics this information can be accessed using exact-WKB techniques.

Abstract - Presentation
Keynote Speaker

Calabi-Yau manifolds: from geometry, to physics, to AI-driven Discovery

Yang-Hui He¹.

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Abstract. Calabi-Yau geometries have been at the fruitful intersection between pure mathematics and theoretical physics. Over the last 8 years, they have also provided a rich playground for AI-driven explorations in scientific discovery, inspiring approaches to problems that arise in diverse areas ranging from field theory, to algebraic geometry, to number theory. We give an overview of this progress, with a perspective on how AI will be transformative in the mathematical sciences in three intertwined directions: theorem-proving, conjecture formulation, and Large Language Models.

Abstract - Presentation
Keynote Speaker

GR from RG

Mohammad Mehdi Sheikh-Jabbari ¹.

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Abstract. We demonstrate how the effective action of a non-gravitating quantum field theory in the ultraviolet (UV) develops an Einstein-Hilbert term in the infrared (IR). Hence, we show how General Relativity (GR) is induced from the RG flow. To this end, we use the holographic AdS/CFT framework, while as we argue this framework is not an essential part of the main proposal and result. Our GR from RG proposal is not only providing a new perspective on the nature of GR and gravity, it also points to a conceptually new addition to the standard Wilsonian RG flow, also provides the mechanism to evade the Weinberg-Witten no-go theorem. The GR from RG proposal also suggests that one should revisit the search for a quantum theory of gravity: gravity is not a fundamental interaction and need not be quantized, the same way that one usually does not quantize fluid dynamics.

Abstract - Presentation
Keynote Speaker

Beyond Krylov Complexity

Mohsen Alishahiha ¹.

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Abstract. We introduce Krylov-space memory cores as stationary, depth-resolved structures revealing how anomalous initial-state memory is organized in otherwise thermalizing nonintegrable systems. Stationary occupation profiles locate late-time probability along the Krylov chain, while complementary diagnostics characterize residual fluctuations, Gibbs mismatch, and persistent current fluctuations. Across weak thermalization, confinement-induced anomalous dynamics, and many-body scarring, anomalous states develop compact low-depth memory cores embedded within broader occupation halos. Reference states lack comparable signal strength and compactness.

Abstract - Presentation
Keynote Speaker

Euclidean Wormholes, Quantum Chaos, and the Factorization Puzzle

Hong Liu ¹.

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Abstract. I propose a resolution of the factorization puzzle in AdS/CFT based on quantum-chaotic behavior in the large- N limit. In this framework, Euclidean wormhole contributions admit a boundary interpretation within a single quantum theory, without invoking an ensemble average.

Full paper - Presentation

Modified Yang-Mills AdS black brane and DC conductivity

Mehdi Sadeghi¹.

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Abstract. In this study, we construct a black brane solution within Einstein-Hilbert gravity with a negative cosmological constant, coupled to a non-Abelian modified Yang-Mills field. We derive the explicit form of this solution and compute the DC conductivity associated with the non-Abelian gauge field via the AdS/CFT correspondence. Our results demonstrate a violation of the conductivity bound for $\gamma > 0$, while naturally reducing to the standard Yang-Mills results as the nonlinear parameter γ approaches zero.

Keywords: Non-abelian color DC Conductivity; Black brane; AdS/CFT duality; Modified Yang-Mills

1 Introduction

Nonlinear modifications of gauge theories have garnered significant interest in theoretical physics due to their rich structure and potential applications in holography, condensed matter systems, and quantum gravity. Among these, the Yang-Mills (YM) theory, which underpins the Standard Model of particle physics, has been extensively studied in both its classical and quantum formulations. In recent years, inspired by developments in nonlinear electrodynamics particularly the ModMax [1] model there has been growing interest in exploring nonlinear generalizations of Yang-Mills theories that preserve desirable symmetries and exhibit novel physical properties. ModMax is the most recent model, and various nonlinear models have been discussed in the literature, including Born-Infeld, arcsin, power-law inverse, and other NEDs [2]-[8].

The ModYang-Mills model, recently introduced as a nonlinear, conformally invariant, and duality-symmetric extension of the Yang-Mills framework, has opened new avenues in the study of nonlinear gauge theories. Unlike the conventional Yang-Mills theory, the ModYang-Mills model retains conformal invariance and duality symmetry, which are fundamental to understanding gauge dualities and the structure of nonlinear gauge theories. Its success has motivated the search for analogous modifications in non-Abelian gauge theories, leading to the formulation of what can be termed Modified Yang-Mills (ModYang-Mills) theories.

Such models extend the Yang-Mills action by incorporating nonlinear functions of the field strength tensor, designed to preserve key symmetries such as gauge invariance and conformal invariances similar to the properties of the ModMax model. These modifications are not only theoretically intriguing but also play a crucial role in holographic setups, where they influence the properties of dual strongly coupled systems, including conductivities and phase transitions.

The Anti-de Sitter/Conformal Field Theory (AdS/CFT) duality [9, 10] is a powerful theoretical framework that establishes a correspondence between gravitational theories in $(d + 1)$ -dimensional Anti-de Sitter (AdS) spacetime and strongly coupled conformal field theories (CFTs) in d dimensions. This holographic principle enables the study of complex quantum many-body systems via classical gravity solutions, providing insights into non-perturbative phenomena. A notable development within this context is the fluid-gravity duality, which relates solutions of Einstein's equations with black branes to relativistic fluid dynamics on the boundary, effectively modeling strongly coupled fluids with universal transport properties. These holographic approaches have been particularly fruitful in calculating transport coefficients such as DC electrical conductivity and thermal conductivity, which are crucial for understanding charge and heat transport in condensed matter systems. By analyzing perturbations in the bulk geometry, one can derive these conductivities in the boundary theory, revealing universal behavior and bounds, such as the Kovtun-Son-Starinets (KSS) bound on viscosity, and shedding light on the nature of strongly correlated materials and quantum critical points.

In this work, we focus on the gravitational dual of a ModYang-Mills theory, specifically exploring the implications of a modified non-Abelian gauge sector within an Anti-de Sitter (AdS) black brane background. Our primary aim is to analyze the DC conductivity in the boundary theory, which provides valuable insights into the transport properties of the dual strongly coupled system. The nonlinearities introduced in the ModYang-Mills sector are expected to lead to novel features in the holographic conductivity, offering potential applications in modeling condensed matter phenomena with nonlinear chromo-responses.

2 Modification of Yang-Mills AdS black brane solution

The four-dimensional action for the non-Abelian modification of Yang-Mills theory in an Anti-de Sitter (AdS) background, incorporating a negative cosmological constant, is expressed as [6, 8]:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2\Lambda + \mathcal{L} \right], \quad (1)$$

where R is the Ricci scalar, and Λ is related to the AdS radius l by $\Lambda = -\frac{3}{l^2}$. The Yang-Mills invariant quantity is given via:

$$\mathcal{F} = \text{Tr}(F_{\mu\nu}^{(a)} F^{(a)\mu\nu}), \quad (2)$$

The trace operation is defined as $\text{Tr}(\cdots) = \sum_{i=1}^3 (\cdots)$. The Yang-Mills field strength tensor for the $SU(2)$ gauge field takes the form:

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} - if_{bc}^a [A_\mu^{(b)}, A_\nu^{(c)}], \quad (3)$$

with the gauge potential $A_\mu^{(a)}$ represented through the diagonal generator of the Cartan subalgebra of $SU(2)$ and f_{bc}^a are the structure constants of the group.

In addition, the Lagrangian density $\mathcal{L}$ corresponds to the modified Yang-Mills (ModYang-Mills) Lagrangian, assumed to be comparable in three- and four-dimensional spacetimes:

$$\mathcal{L} = -\mathcal{F} \cosh \gamma + \sqrt{\mathcal{F}^2 + \tilde{\mathcal{F}}^2} \sinh \gamma, \quad (4)$$

where γ is a dimensionless parameter termed the ModYang-Mills parameter.

In this formulation, $\mathcal{F} = \text{Tr}(F_{\mu\nu}^{(a)} F^{(a)\mu\nu})$ is the scalar invariant, and $\tilde{\mathcal{F}} = F_{\mu\nu}^{(a)} \tilde{F}^{(a)\mu\nu}$ is the pseudoscalar, with

$$\tilde{F}^{(a)\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda}^{(a)}. \quad (5)$$

Notably, when $\gamma = 0$, the ModYang-Mills Lagrangian reduces to the conventional linear Yang-Mills form: $\mathcal{L} = -\mathcal{F}$. As $\gamma \rightarrow 0$, the nonlinearity diminishes and the theory approaches the standard Yang-Mills configuration [11].

Variation of the action (1) with respect to the metric $g_{\mu\nu}$ and gauge fields $A_\nu^{(a)}$ gives the equations of motion:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} - 2e^{-\gamma} F_\mu^{(a)\alpha} F_{\nu\alpha}^{(a)} + \frac{1}{2} e^{-\gamma} F_{\alpha\beta}^{(a)} F^{(a)\alpha\beta} g_{\mu\nu} = 0, \quad (6)$$

$$\nabla_\mu (e^{-\gamma} F^{(a)\mu\nu}) = 0. \quad (7)$$

Our goal is to construct an asymptotically AdS black brane solution in a four-dimensional, flat, symmetric spacetime. To this end, a static metric ansatz of the form:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{l^2}(dx^2 + dy^2), \quad (8)$$

is adopted, where $f(r)$ is an unknown function of the radial coordinate r , and l is related to the cosmological constant.

The gauge field $A_\mu^{(a)}$ is expressed via the diagonal generator of $SU(2)$, represented as:

$$\mathbf{A}^{(a)} = \frac{i}{\sqrt{2}} h(r) dt \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (9)$$

Substituting into the gauge equations, we obtain a differential equation for $h(r)$:

$$2h'(r) + rh''(r) = 0, \quad (10)$$

whose solution is:

$$h(r) = C_2 - \frac{C_1}{r}, \quad (11)$$

with C_1 and C_2 being integration constants that are related to the boundary chemical potential. Regularity at the horizon, $r = r_h$, imposes $A_t(r_h) = 0$, leading to:

$$C_2 = \frac{C_1}{r_h}. \quad (12)$$

Consequently, the gauge potential reads:

$$h(r) = Q \left(\frac{1}{r} - \frac{1}{r_h} \right), \quad (13)$$

where Q characterizes the charge related to the gauge field.

The invariant scalar $\mathcal{F}$ is obtained via:

$$F_{tr}^{(a)} = -F_{rt}^{(a)} = \frac{iQ}{\sqrt{2}r^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (14)$$

When $\gamma = 0$, this solution reduces to the classic non-Abelian Yang-Mills black brane. Considering the rr -component of the Einstein equations yields:

$$rf'(r) + f(r) + \Lambda r^2 + e^{-\gamma} r^2 h'(r)^2 = 0, \quad (15)$$

which integrates to give:

$$f(r) = -\frac{2m}{r} - \frac{\Lambda r^2}{3} + \frac{Q^2}{r^2} e^{-\gamma}, \quad (16)$$

where m is an integration constant associated with the black brane mass.

The event horizon, located at $r = r_h$, satisfies $f(r_h) = 0$, fixing m as:

$$m = \frac{r_h^3}{2l^2} + \frac{Q^2}{2r_h} e^{-\gamma}. \quad (17)$$

Thus, the metric function becomes:

$$f(r) = \frac{r^2}{l^2} \left(1 - \frac{r_h^3}{r^3}\right) + \frac{Q^2}{r^2} \left(1 - \frac{r}{r_h}\right) e^{-\gamma}. \quad (18)$$

The Hawking temperature, associated with the surface gravity at the horizon, is given by:

$$T = \frac{f'(r_h)}{4\pi} = \frac{3r_h}{4\pi l^2} - \frac{Q^2}{4\pi r_h^3} e^{-\gamma}. \quad (19)$$

3 Color non-abelian DC conductivity

To compute the retarded Green's function $\langle J^\mu(\omega) J^\nu(-\omega) \rangle$, we perturb the gauge field as $A_\mu \rightarrow A_\mu + \tilde{A}_\mu$ and expand the action up to second order in the perturbation $\tilde{A}_\mu$ [12]–[22]. The boundary current J_a^μ can be identified from the perturbed part of the action as follows:

$$S \rightarrow S + \int d^4x A_\mu^a J_a^\mu. \quad (20)$$

The retarded Green's function is obtained via the AdS/CFT correspondence:

$$G_{\mu\nu}^{(ab)}(k_\mu) = \langle J_\mu^{(a)}(\omega) J_\nu^{(b)}(-\omega) \rangle = \frac{\delta^2 S}{\delta \tilde{A}_{(a)}^{0\mu} \delta \tilde{A}_{(b)}^{0\nu}}, \quad (21)$$

where $\tilde{A}_\nu^0$ denotes the boundary value of the gauge field. The non-Abelian color DC conductivity can be computed using the Green-Kubo formula:

$$\sigma_{\mu\nu}^{(ab)}(k_\mu) = -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \Im G_{\mu\nu}^{(ab)}(k_\mu), \quad (22)$$

which indicates that DC conductivities are related to the retarded Green's function of the boundary current J^i . The conductivities along the i and j directions are expressed as:

$$\sigma_{\mu\nu}^{(ab)}(k_\mu)(x, y) = \frac{\delta J_{(a)}^\mu(x)}{\delta E_{(b)}^\nu(y)}. \quad (23)$$

Due to rotational symmetry in the xy plane, the color DC conductivity is a scalar quantity:

$$\sigma_{(ab)}^{ij} = \sigma_{(ab)} \delta^{ij}. \quad (24)$$

Next, we consider the perturbative part of the gauge field as $\tilde{A}_x = \tilde{A}_x(r)e^{-i\omega t}$, with small ω , ensuring the hydrodynamic regime. Substituting this into the action (see Eq. (1)) and retaining terms up to second order in $\tilde{A}$, yields:

$$S^{(2)} = - \int d^4x \frac{e^{-\gamma}}{f} \left[-4f^2 \left((\partial_r \tilde{A}_x^{(1)})^2 + (\partial_r \tilde{A}_x^{(2)})^2 + (\partial_r \tilde{A}_x^{(3)})^2 \right) + \left(\tilde{A}_x^{(1)2} + \tilde{A}_x^{(2)2} \right) (4\omega^2 - 8h(r)^2) + 4\omega^2 \tilde{A}_x^{(3)2} \right]. \quad (25)$$

Variation of $S^{(2)}$ with respect to $\tilde{A}_x^a$ provides the equations of motion:

$$f(f\tilde{A}_x^{(1)'})' + (\omega^2 - 2h(r)^2)\tilde{A}_x^{(1)} = 0, \quad (26)$$

$$f(f\tilde{A}_x^{(2)'})' + (\omega^2 - 2h(r)^2)\tilde{A}_x^{(2)} = 0, \quad (27)$$

$$f\left(f\tilde{A}_x^{(3)'}\right)' + \omega^2\tilde{A}_x^{(3)} = 0. \quad (28)$$

To begin with, we analyze Eqs. (26), (27), and (28) near the event horizon. By expanding these equations close to the horizon and utilizing the relation $f \sim 4\pi T(r - r_h)$, derived from the Hawking temperature, we can express the solutions for $\tilde{A}_x^{(a)}$ as:

$$\tilde{A}_x^{(a)} \sim (r - r_h)^{z_a}, \quad a = 1, 2, 3, \quad (29)$$

where

$$z_1 = z_2 = \pm i \frac{\sqrt{\omega^2 - 2h(r_h)^2}}{4\pi T}, \quad (30)$$

$$z_3 = \pm i \frac{\omega}{4\pi T}. \quad (31)$$

To solve Eq. (26), (27), and (28), we employ the following ansätze:

$$\tilde{A}_x^{(1)} = \tilde{A}_\infty^{(1)} \left(\frac{-3}{\Lambda r^2} f \right)^{z_1} (1 + i\omega h_1(r) + \dots), \quad (32)$$

$$\tilde{A}_x^{(2)} = \tilde{A}_\infty^{(2)} \left(\frac{-3}{\Lambda r^2} f \right)^{z_2} (1 + i\omega h_2(r) + \dots), \quad (33)$$

$$\tilde{A}_x^{(3)} = \tilde{A}_\infty^{(3)} \left(\frac{-3}{\Lambda r^2} f \right)^{z_3} (1 + i\omega h_3(r) + \dots), \quad (34)$$

where $\tilde{A}_\infty^{(a)}$ denote the boundary values of the fields, and z_i correspond to the negative signs in Eqs. (30) and (31).

Next, substituting Eq. (34) into Eq. (28) and focusing on the linear order in ω , we obtain:

$$\begin{aligned} & 4\sqrt{2}\pi r^2 T f(r) \sqrt{-h(r_h)^2} f'(r) h_1'(r) + 2\sqrt{2}\pi r^2 T f(r) \sqrt{-h(r_h)^2} h_1(r) f''(r) \\ & - i r^2 h(r_h)^2 h_1(r) f'(r)^2 - 4\sqrt{2}\pi r T f(r) \sqrt{-h(r_h)^2} h_1(r) f'(r) \\ & + 4i r f(r) h(r_h)^2 h_1(r) f'(r) - 8i\pi^2 r^2 T^2 f(r) f'(r) h_1'(r) \\ & - 8\sqrt{2}\pi r T f(r)^2 \sqrt{-h(r_h)^2} h_1'(r) + 4\sqrt{2}\pi T f(r)^2 \sqrt{-h(r_h)^2} h_1(r) \\ & - 4i f(r)^2 h(r_h)^2 h_1(r) - 8i\pi^2 r^2 T^2 f(r)^2 h_1''(r) + 16i\pi^2 r^2 T^2 h(r)^2 h_1(r) = 0, \end{aligned} \quad (35)$$

along with

$$r (f'(r)(r h_3'(r) - 2) + r f''(r)) + f(r) (r^2 h_3''(r) + 2) = 0. \quad (36)$$

The form of the solution for $h_3(r)$ is given by:

$$h_3(r) = C_3 + \int^r \left(\frac{2}{u} + \frac{C_4 - f'(u)}{f(u)} \right) du, \quad (37)$$

where C_3 and C_4 are constants of integration. Near the horizon, the behavior of $h_3(r)$ can be approximated as:

$$h_3(r) \approx \left(\frac{C_4}{4\pi T} - 1 \right) \log(r - r_h) + \text{finite terms}. \quad (38)$$

The value of C_4 is determined by requiring regularity at the horizon, which yields:

$$C_4 = 4\pi T. \quad (39)$$

By substituting the solution for $\tilde{A}_x^{(3)}$ into Eq. (25) and varying with respect to $\tilde{A}_\infty^{(3)}$, we can extract the Green's function:

$$G_{xx}^{(33)}(\omega, \vec{0}) = e^{-\gamma} \left(\tilde{A}_x^{(3)} \right)^* f(r) \partial_r \tilde{A}_x^{(3)} \Big|_{r \rightarrow \infty} = -i \frac{\omega C_4 e^{-\gamma}}{4\pi T}. \quad (40)$$

Substituting $f(r)$, $h_3(r)$, and $h'(r)$ into this expression, and using Eq. (22), we find:

$$\sigma_{xx}^{(33)} = - \lim_{\omega \rightarrow 0} \frac{1}{\omega} \Im G_{xx}^{(33)}(\omega, \vec{0}) = e^{-\gamma}. \quad (41)$$

This indicates that the conductivity bound is violated for this model for the $\gamma > 0$ and saturates the bound for $\gamma = 0$ [23]:

$$\sigma_{xx}^{(33)} = 1. \quad (42)$$

The values of $\sigma_{xx}^{(11)}$ and $\sigma_{xx}^{(22)}$ can be obtained via the same method as for $\sigma_{xx}^{(33)}$, resulting in:

$$\sigma_{xx}^{(11)} = \sigma_{xx}^{(22)} = 0. \quad (43)$$

This suggests that the non-Abelian DC conductivity, with respect to the color indices, is diagonal, and Ohm's law maintains a diagonal form within this framework. Furthermore, if the gauge field is considered along the σ_1 or σ_2 directions (as outlined in 9), the conductivity in those directions will be non-zero.

4 Conclusions

In this work, we constructed an asymptotically AdS black brane solution within Einstein-Hilbert gravity coupled to a non-Abelian modified Yang-Mills (ModYM) gauge field. In the limit $\gamma \rightarrow 0$, the solution reduces to the standard non-Abelian Yang-Mills black brane, consistently recovering known results.

Using the AdS/CFT correspondence, we computed the color DC conductivity by analyzing quadratic perturbations of the gauge field in the bulk. The holographic calculation yields a diagonal conductivity tensor in color space, with the non-vanishing component

$$\sigma_{xx}^{(33)} = e^{-\gamma}, \quad (44)$$

while the transverse color components vanish:

$$\sigma_{xx}^{(11)} = \sigma_{xx}^{(22)} = 0. \quad (45)$$

This result has direct implications for the conjectured universal lower bound on conductivity. For the standard Yang-Mills case ($\gamma = 0$), the bound is precisely saturated, $\sigma = 1$, in agreement with previous findings. However, for $\gamma > 0$, the conductivity falls below unity, $\sigma < 1$, indicating a violation of the bound within this modified non-Abelian framework. Conversely, for $\gamma < 0$, the conductivity exceeds the bound, $\sigma > 1$.

Although in our ModYM model the conductivity remains finite ($\sigma = e^{-\gamma} > 0$) for any finite γ , violating the bound $\sigma \geq 1$ removes the universal metallic floor. This opens the door for a genuine metal-insulator transition, such as a *Mott-like* transition, in more general setups. Our results indicate that as γ increases, the system departs from the metallic state and approaches an insulating phase, even though the conductivity never strictly vanishes in this particular model.

The present analysis opens several avenues for future investigation. It would be interesting to extend this study to include momentum dissipation (e.g., via axion fields or spatially modulated sources) to explore the full frequency-dependent optical conductivity and possible phase transitions. Furthermore, embedding this ModYM sector into more general higher-derivative gravity or exploring its effects on the holographic entanglement entropy could provide deeper insight into the role of nonlinear non-Abelian gauge symmetries in strongly coupled systems.

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Abstract - Presentation

Hadronic light-by-light scattering and the muon $g - 2$ from AdS/QCD

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Abstract. Hadronic light-by-light (HLbL) scattering is a rich and complex QCD process and a key input in the Standard Model computation of the anomalous magnetic moment of the muon. For an accurate determination of the latter it is necessary to understand HLbL scattering for a wide range of energies in a quantitative manner. In this talk I will present an analysis of HLbL scattering and the muon $g - 2$ within holographic models of QCD (hQCD) and compare the results to phenomenological dispersive approaches. I will highlight in particular the high energy constraints which follow from the operator product expansion in perturbative QCD and show that the infinite towers of axial vector and tensor meson resonances in hQCD are crucial ingredients in implementing these constraints. Finally I will show that tensor mesons, as described in hQCD, can account for the current discrepancy between dispersive and lattice studies on the HLbL contribution to the muon $g - 2$.

Keywords: holography, QCD, phenomenology

Full paper - Presentation

Emergent Anti-Gravity from Entropic Forces on Gpc-Scale: A Holographic Origin of Dynamical Dark Energy

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Abstract. We propose that cosmic acceleration originates from entropic forces on Gpc-scale void boundaries, extending Verlinde’s holographic gravity to cosmology. Hierarchical void merging reduces surface tension, generating a repulsive pressure. Starting from the Tsallis nonextensive entropy on the Hubble horizon, we derive a first-order logarithmic equation of state. From this, a quadratic approximation the Quadratic Dark Energy Model (QDEM) with equation of state $p = w\rho - \beta\rho^2$ is obtained as a controlled local late-time expansion near the present-day density. Remarkably, the model naturally predicts an effective equation of state $w \simeq -1$ with small deviations controlled by the entropy parameter ϵ , in excellent agreement with current observational constraints from CMB, BAO, and SNIa. The QDEM features a global de Sitter attractor $\rho^* = (1 + w)/\beta$, which reframes the fine-tuning and coincidence problems, and satisfies the maximum entropy condition $S''(z) < 0$ at late times, ensuring thermodynamic consistency with holographic equipartition. The framework unifies small-scale gravity and cosmic anti-gravity as emergent entropic phenomena, offering a physical alternative to the cosmological constant. Preliminary CMB, BAO, and SNIa data show qualitative agreement and suggest potential alleviation of the Hubble and S_8 tensions.

Keywords: Dynamical Dark Energy; Entropic Gravity; Holographic Principle; Cosmic Voids; Tsallis Entropy

1 Introduction and Motivations

The discovery of the accelerated expansion of the Universe from Type Ia supernovae [1, 2] established dark energy as the dominant component of the cosmic energy budget, constituting approximately 70% of the total energy density today. The simplest and most successful explanation to date is the cosmological constant Λ , with equation of state $w = p/\rho = -1$, which provides an excellent fit to a wide range of observations including cosmic microwave background (CMB) anisotropies, large-scale structure surveys, and baryon acoustic oscillations (BAO) [3]. Despite its empirical success, the Λ CDM model faces profound theoretical challenges: the fine-tuning problem—the observed value of $\rho_\Lambda \sim (10^{-3} \text{ eV})^4$ is 10^{120} orders of

magnitude smaller than the natural scale of vacuum energy predicted by quantum field theory [4]—and the coincidence problem, which questions why dark energy and matter densities are comparable precisely today [5]. These theoretical puzzles are now accompanied by growing observational tensions, most notably the H_0 tension ($\sim 5\sigma$) between local measurements and Planck CMB inference [6, 3] and the S_8 tension between CMB-inferred and weak-lensing measurements of the amplitude of matter fluctuations [7, 8]. Recent results from the Dark Energy Spectroscopic Instrument (DESI) DR2 suggest a preference for dynamical dark energy [9, 10], further motivating the exploration of alternatives to the standard paradigm.

In this work, we propose a novel framework that connects the holographic principle, entropic gravity, and the thermodynamics of cosmic voids to provide a physical origin for dynamical dark energy. Our central thesis is that **anti-gravity at cosmological scales is an emergent entropic phenomenon**, arising from the thermodynamic evolution of the boundaries of Gpc-scale cosmic voids. This extends Verlinde’s entropic gravity paradigm [11]—where gravity emerges as an entropic force due to information changes on holographic screens—to the largest scales of the cosmic web. As cosmic voids merge hierarchically (see Fig. 1 for a schematic illustration), their boundary surface tension decreases, generating a repulsive entropic pressure that drives the accelerated expansion of the Universe. This mechanism is captured by the Quadratic Dark Energy Model (QDEM) [12, 13, 14, 15], defined by Eq. (35). The QDEM is derived as a controlled local late-time approximation to a logarithmic equation of state that follows from the nonextensive Tsallis entropy on the Hubble horizon [16, 17], providing a direct connection to the holographic principle. A key feature of the QDEM is that it naturally yields an equation of state parameter $w \simeq -1$, with deviations controlled solely by the small parameter $\epsilon = \delta - 1$ and the entropy normalization parameter a_1 . This is in contrast to phenomenological parametrizations such as CPL, where w_0 and w_a are free parameters without a clear physical origin. The proximity of w to -1 is therefore a prediction of the model, not an input assumption, and aligns well with current observational bounds.

The QDEM features a global de Sitter attractor $\rho^* = (1 + w)/\beta$ that naturally reframes the fine-tuning and coincidence problems, while exhibiting asymptotic approach to the phantom divide without actual crossing. The model satisfies the maximum entropy condition $S''(z) < 0$ at late times [14], indicating a well-behaved thermodynamic evolution toward equilibrium—consistent with the holographic equipartition of degrees of freedom on the cosmic horizon [18, 19]. Preliminary observational constraints from CMB, BAO, and supernovae show qualitative consistency with the model and suggest potential alleviation of the Hubble and S_8 tensions [20, 21].

This paper is organized as follows. In Sec. 2, we establish the holographic foundation of the model, deriving a logarithmic equation of state from Tsallis entropy on the Hubble horizon, and then obtaining the quadratic equation of state as a local late-time approximation. In Sec. 3, we present the QDEM framework, including its analytical solution and asymptotic behavior. In Sec. 4.1, we discuss the entropic origin of cosmic acceleration, incorporating the void hierarchy and configuration entropy arguments. In Sec. 4.2, we discuss perturbation stability and effective sound speed. In Sec. 4.3, we discuss observational signatures and comparison with data. In Sec. 4.5, we summarize limitations and outline future work, including MCMC analyses.

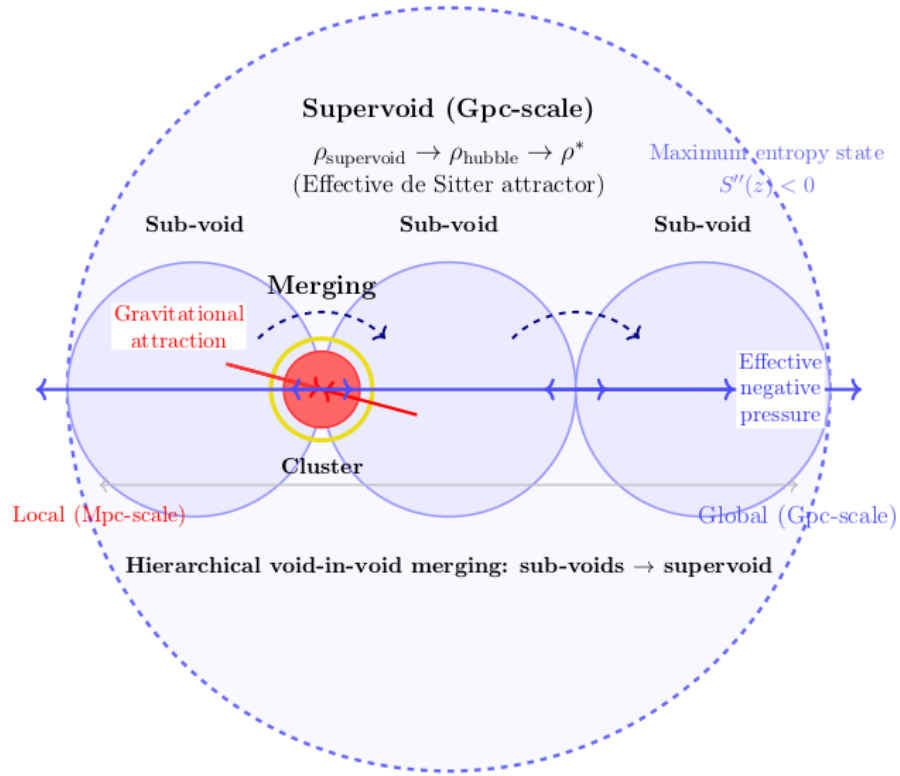


Figure 1: **Hierarchical void merging and the emergence of cosmic acceleration.** Sub-voids at local scales (Mpc) are nested within a larger supervoid at global scales (Gpc). The blue arrows represent the repulsive entropic pressure generated by void merging. The density of the Gpc-void is marked by $\rho \rightarrow \rho^*$, indicating the approach to the de Sitter attractor density where the effective equation of state becomes $w_{\text{eff}} \rightarrow -1$.

2 Holographic Foundation of the Quadratic EoS

2.1 Thermodynamics of the Hubble Horizon

We consider a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe with scale factor $a(t)$ and Hubble parameter $H = \dot{a}/a$. We use the Hubble radius

$$R_h = \frac{c}{H}, \quad (1)$$

as the characteristic cosmological length scale. The associated area and volume are

$$A = 4\pi R_h^2 = \frac{4\pi c^2}{H^2}, \quad (2)$$

and

$$V = \frac{4\pi}{3} R_h^3 = \frac{4\pi c^3}{3H^3}. \quad (3)$$

The use of a thermal description for the Hubble sphere requires an additional quasi-de Sitter approximation. In the regime in which the time variation of the horizon is sufficiently slow, we adopt the effective horizon temperature [22],

$$T_h \simeq \frac{\hbar H}{2\pi k_B}. \quad (4)$$

Equation (4) is therefore used here as an effective quasi-de Sitter temperature, rather than as a general exact temperature for an arbitrary dynamical FLRW horizon.

2.2 Tsallis Nonextensive Entropy

The Bekenstein–Hawking entropy is

$$S_{\text{BH}} = \frac{k_{\text{B}} A}{4\ell_{\text{P}}^2}, \quad \ell_{\text{P}}^2 = \frac{G\hbar}{c^3}. \quad (5)$$

Motivated by the nonadditive statistical mechanics of systems with long-range correlations, we consider the following effective power-law generalization,

$$S_T = \frac{k_{\text{B}}}{4} \left(\frac{A}{\ell_{\text{P}}^2} \right)^\delta, \quad (6)$$

where δ measures the departure from the extensive Bekenstein–Hawking form. The normalization in Eq. (6) is chosen such that

$$S_T|_{\delta=1} = S_{\text{BH}}. \quad (7)$$

This parametrization should be regarded as an effective entropy ansatz for the cosmological horizon. The use of a non-Boltzmann entropy is motivated by the nonextensive statistical-mechanical literature, but its application to the Hubble horizon does not follow uniquely from that literature [16, 17].

For later convenience, Eq. (6) can be written as

$$S_T = \gamma(\delta) A^\delta, \quad \gamma(\delta) = \frac{k_{\text{B}}}{4\ell_{\text{P}}^{2\delta}}. \quad (8)$$

2.3 Effective Entropic Pressure

The thermodynamic relation between pressure, entropy and volume must be specified as an additional assumption. It is not, in general, a fundamental identity that follows directly from $dE = T dS - P dV$. In the present effective description we therefore introduce the entropic contribution to the pressure through

$$P_{\text{ent}} = -\chi T_h \frac{dS_T}{dV}, \quad (9)$$

where χ is a dimensionless normalization parameter.

The factor χ is introduced explicitly because the direct identification $\chi = 1$ does not reproduce the de Sitter equation of state in the extensive Bekenstein–Hawking limit. We choose

$$\chi = \frac{3}{2}, \quad (10)$$

so that, for $\delta = 1$, the present effective construction gives $P_{\text{ent}} = -\rho$ when combined with the flat Friedmann equation. This is a calibration of the effective pressure prescription and is not an independent consequence of the Tsallis entropy itself.

From Eq. (6),

$$\frac{dS_T}{dA} = \gamma(\delta) \delta A^{\delta-1}, \quad (11)$$

while Eqs. (2) and (3) give

$$\frac{dA}{dV} = \frac{2}{R_h}. \quad (12)$$

Hence

$$\frac{dS_T}{dV} = \frac{2\gamma(\delta)\delta}{R_h} A^{\delta-1}. \quad (13)$$

Substituting Eqs. (4) and (13) into Eq. (9), we obtain

$$\begin{aligned} P_{\text{ent}} &= -\chi \frac{\hbar H}{2\pi k_B} \frac{2\gamma(\delta)\delta}{R_h} A^{\delta-1} \\ &= -\frac{\chi \hbar H}{\pi k_B R_h} \gamma(\delta)\delta A^{\delta-1}. \end{aligned} \quad (14)$$

Using $R_h = c/H$,

$$P_{\text{ent}} = -\frac{\chi \hbar}{\pi c k_B} \gamma(\delta)\delta H^2 A^{\delta-1}. \quad (15)$$

Notice that the factor k_B^{-1} in Eqs. (14) and (15) is required by the temperature normalization.

2.4 Expressing the Pressure in Terms of Energy Density

For a spatially flat universe, the first Friedmann equation is

$$H^2 = \frac{8\pi G}{3c^2} \rho, \quad (16)$$

where ρ denotes the total energy density. Consequently,

$$A = \frac{4\pi c^2}{H^2} = \frac{3c^4}{2G\rho}. \quad (17)$$

Substituting Eqs. (8), (16), and (17) into Eq. (15) gives

$$P_{\text{ent}}(\rho) = -\frac{2\chi}{3} \delta \left(\frac{3c^4}{2G\ell_P^2} \right)^{\delta-1} \rho^{2-\delta}. \quad (18)$$

We therefore define

$$\mathcal{A}_\delta \equiv \frac{2\chi}{3} \delta \left(\frac{3c^4}{2G\ell_P^2} \right)^{\delta-1}, \quad (19)$$

so that

$$P_{\text{ent}}(\rho) = -\mathcal{A}_\delta \rho^{2-\delta}. \quad (20)$$

The dimensions of $\mathcal{A}_\delta$ are $[\rho]^{\delta-1}$, as required by Eq. (20).

As a useful consistency check, in the extensive limit $\delta = 1$, Eq. (20) becomes

$$P_{\text{ent}} = -\frac{2\chi}{3} \rho. \quad (21)$$

With the calibration $\chi = 3/2$ of Eq. (10),

$$P_{\text{ent}}|_{\delta=1} = -\rho, \quad (22)$$

and hence

$$w_{\text{ent}}|_{\delta=1} = -1. \quad (23)$$

This consistency condition fixes the normalization used in the effective pressure prescription.

2.5 Expansion Around the Extensive Limit

We now consider a small departure from the Bekenstein–Hawking limit,

$$\delta = 1 + \epsilon, \quad |\epsilon| \ll 1. \quad (24)$$

Because $\mathcal{A}_\delta$ has δ -dependent dimensions, it is preferable to introduce a dimensionless coefficient before performing the expansion. Let ρ_0 be a reference density, chosen here to be the present-day dark-energy density, and define

$$\mathfrak{A}(\delta) \equiv \mathcal{A}_\delta \rho_0^{1-\delta}. \quad (25)$$

Then

$$P_{\text{ent}}(\rho) = -\rho \mathfrak{A}(1 + \epsilon) \left(\frac{\rho}{\rho_0} \right)^{-\epsilon}. \quad (26)$$

With $\chi = 3/2$ and the normalization in Eq. (6),

$$\mathfrak{A}(1) = 1. \quad (27)$$

We write

$$\mathfrak{A}(1 + \epsilon) = 1 + a_1 \epsilon + O(\epsilon^2), \quad (28)$$

where a_1 is a dimensionless effective parameter that encodes the details of the entropy normalization. Its precise value would require a more complete microscopic model of the horizon degrees of freedom; in the present phenomenological approach, a_1 should be treated as a parameter to be constrained by observational data.

The second factor in Eq. (26) is expanded as

$$\left(\frac{\rho}{\rho_0} \right)^{-\epsilon} = \exp \left[-\epsilon \ln \left(\frac{\rho}{\rho_0} \right) \right] = 1 - \epsilon \ln \left(\frac{\rho}{\rho_0} \right) + O(\epsilon^2). \quad (29)$$

Thus, to first order in ϵ ,

$$P_{\text{ent}}(\rho) = -\rho - \epsilon a_1 \rho + \epsilon \rho \ln \left(\frac{\rho}{\rho_0} \right) + O(\epsilon^2). \quad (30)$$

Equation (30) is the controlled first-order equation of state generated by the present horizon-entropy ansatz. Importantly, the correction produced directly by the Tsallis deformation is logarithmic rather than exactly quadratic. This logarithmic form represents the exact entropic prediction of the model within the first-order expansion in ϵ .

2.6 Local Quadratic Approximation

A quadratic equation of state can nevertheless be obtained as a local late-time approximation near $\rho = \rho_0$. Define

$$\Delta \equiv \frac{\rho - \rho_0}{\rho_0}, \quad |\Delta| \ll 1. \quad (31)$$

The logarithm can then be expanded as

$$\ln\left(\frac{\rho}{\rho_0}\right) = \Delta - \frac{\Delta^2}{2} + O(\Delta^3). \quad (32)$$

To obtain a quadratic equation of state consistently, we retain only the leading term in Δ ,

$$\ln\left(\frac{\rho}{\rho_0}\right) = \frac{\rho - \rho_0}{\rho_0} + O(\Delta^2). \quad (33)$$

Substitution into Eq. (30) gives

$$\begin{aligned} P_{\text{ent}}(\rho) &= -\rho - \epsilon a_1 \rho + \epsilon \rho \frac{\rho - \rho_0}{\rho_0} + O(\epsilon^2, \epsilon \Delta^2) \\ &= -[1 + \epsilon(1 + a_1)]\rho + \frac{\epsilon}{\rho_0} \rho^2 + O(\epsilon^2, \epsilon \Delta^2). \end{aligned} \quad (34)$$

We therefore identify the local quadratic equation of state as

$$P(\rho) = w\rho - \beta\rho^2, \quad (35)$$

with

$$w = -1 - \epsilon(1 + a_1) + O(\epsilon^2), \quad (36)$$

and

$$\beta = -\frac{\epsilon}{\rho_0} + O(\epsilon^2). \quad (37)$$

It is worth emphasizing that the resulting equation of state parameter Eq. (36) is naturally close to -1 for small $|\epsilon|$. This is a direct consequence of the Tsallis entropy expansion around the Bekenstein–Hawking limit, and it provides a physical explanation for why the dark energy equation of state is observed to be so close to -1 today, without invoking a cosmological constant or fine-tuning. The deviation from -1 is controlled by the same parameters (ϵ and a_1) that govern the nonlinear quadratic correction, thereby linking the observed value of w to the microscopic properties of the horizon entropy. This stands in contrast to phenomenological parametrizations such as CPL, where the proximity to -1 is not theoretically motivated but rather imposed by data fitting.

For a positive quadratic coefficient,

$$\beta > 0, \quad (38)$$

the present approximation therefore requires

$$\epsilon < 0, \quad \delta < 1, \quad (39)$$

provided $\rho_0 > 0$. This sign follows from the local expansion and does not by itself establish a microscopic preference for sub-extensive values of δ ; such a preference would require an additional microscopic model of the horizon degrees of freedom.

It is important to stress the regime of validity of Eq. (35). The quadratic form is a local approximation valid for

$$|\epsilon| \ll 1, \quad \left| \frac{\rho - \rho_0}{\rho_0} \right| \ll 1. \quad (40)$$

It is worth emphasizing that the choice of ρ_0 (the present-day dark energy density) as the expansion point is not arbitrary. While a Taylor expansion could, in principle, be performed around any reference density ρ_{ref} within the domain of validity of the power-law equation of state (20), such an expansion would only be accurate in the immediate vicinity of that specific point. The selection of ρ_0 is therefore physically motivated: the QDEM is designed as an effective model for the recent cosmic epoch, where observational data (CMB, BAO, SNIa, and local H_0 measurements) are most precise and where the accelerated expansion is most prominent. Expanding around ρ_0 ensures that the quadratic approximation (35) is optimally accurate in the redshift range that is most relevant for dark energy phenomenology. This does not imply that the model is invalid at higher redshifts; rather, for epochs far from the present ($\rho \ll \rho_0$ or $\rho \gg \rho_0$), the more fundamental logarithmic equation of state (30), or equivalently the original power-law form (20), should be employed. Thus, the QDEM is a controlled local late-time approximation, and its validity is explicitly restricted to the regime stated in Eq. (40). This is a standard and well-established practice in effective field theory, where a simpler description is used in a restricted domain while a more complete theory is available for the full range of parameters.

The restriction to small deviations from extensivity, $|\epsilon| \ll 1$, is not merely a mathematical convenience but a physical necessity. Recent studies have shown that even mild departures from $\delta = 1$ lead to pathological behavior in the effective dark energy sector, such as negative or complex energy densities, divergent equations of state, or an unacceptably large early-time contribution that spoils radiation domination and violates BBN and CMB constraints [27]. Consequently, a viable cosmology based on Tsallis entropy emerges only in the extensive limit, $\delta = 1$, effectively reducing the model to Λ CDM [27]. This underscores that the smallness of ϵ in our framework is an essential requirement for cosmological consistency, not an ad-hoc assumption.

2.7 Holographic Interpretation and Emergent Gravity

The preceding derivation provides an effective connection between a generalized horizon entropy and an equation of state. It should be distinguished from the separate holographic-equipartition framework developed by Padmanabhan [18, 19] and the entropic-gravity proposal of Verlinde [11].

In the emergent-space description, the expansion of a spatially flat FLRW universe can be expressed in terms of the difference between the surface and bulk degrees of freedom,

$$\frac{dV}{dt} = c \ell_P^2 (N_{\text{sur}} - N_{\text{bulk}}), \quad (41)$$

where the explicit factor of c restores SI dimensions. For the Bekenstein–Hawking entropy one may write

$$N_{\text{sur}} = \frac{A}{\ell_P^2}, \quad (42)$$

while the bulk degrees of freedom are associated with the Komar active gravitational energy,

$$E_K = (\rho + 3P)V, \quad (43)$$

through

$$N_{\text{bulk}} = -\frac{2E_K}{k_B T_h}. \quad (44)$$

For a de Sitter state with $P = -\rho$, one has $N_{\text{sur}} = N_{\text{bulk}}$, corresponding to holographic equipartition.

A generalized entropy such as Eq. (6) suggests the effective replacement

$$N_{\text{sur}} \longrightarrow \frac{4S_T}{k_B}, \quad (45)$$

but the complete emergent-space dynamics must then be rederived consistently from the modified surface degrees of freedom. We therefore do not identify the phenomenological pressure relation Eq. (9) with a direct derivation of Eq. (41); rather, the two constructions provide complementary thermodynamic interpretations of the same holographic framework.

The present result also clarifies the role of the quadratic equation of state. The exact entropy-based result is the power law Eq. (20), which yields the logarithmic equation of state Eq. (30) as a first-order expansion in ϵ . The QDEM form in Eq. (35) is a controlled local quadratic approximation near a chosen reference density. A microscopic derivation connecting this correction to the surface tension and hierarchical merging of cosmic voids remains an additional physical ingredient of the model [12, 13].

3 The Quadratic Dynamical Dark Energy Model Framework

The QDEM is defined by the equation of state in Eq. (35) with $\beta > 0$. It is important to emphasize that this quadratic form is derived as a local approximation to the more fundamental logarithmic equation of state Eq. (30), valid under the conditions stated in Eq. (40). Therefore, the QDEM should be understood as an effective late-time model applicable in the redshift range where $\rho(z)$ remains sufficiently close to ρ_0 . For epochs far from the present (large z), the full logarithmic or power-law form should be employed.

The conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

yields the energy density evolution. Writing $p = w\rho - \beta\rho^2$ and using $d\rho/dt = -3H(\rho + p)$, we obtain

$$\frac{d\rho}{d \ln a} = -3(\rho + p) = -3[(1 + w)\rho - \beta\rho^2].$$

This first-order nonlinear differential equation can be integrated to give

$$\rho(z) = \rho_0 \left[\frac{w_a}{1 + w} + \left(1 - \frac{w_a}{1 + w} \right) (1 + z)^{-3(1+w)} \right]^{-1}, \quad (46)$$

where $w_a \equiv \beta\rho_0$ and ρ_0 is the present-day dark energy density.

The effective equation of state is obtained by substituting Eq. (46) into

$$w_{\text{eff}}(z) = \frac{p}{\rho} = w - \beta \frac{\rho(z)}{\rho_0},$$

leading to

$$w_{\text{QDEM}}(z) = w_0 + w_a - \frac{w_a(1 + w_0 + w_a)}{(1 + w_0)(1 + z)^{-3(1+w_0+w_a)} + w_a}, \quad (47)$$

with $w_0 = w(z = 0) = w - w_a$.

The asymptotic limits of Eq. (47) are:

$$\lim_{z \rightarrow \infty} w_{\text{QDEM}}(z) = \begin{cases} -1, & 1 + w > 0, \\ w, & 1 + w < 0, \end{cases} \quad \lim_{z \rightarrow -1} w_{\text{QDEM}}(z) = \begin{cases} w, & 1 + w > 0, \\ -1, & 1 + w < 0. \end{cases} \quad (48)$$

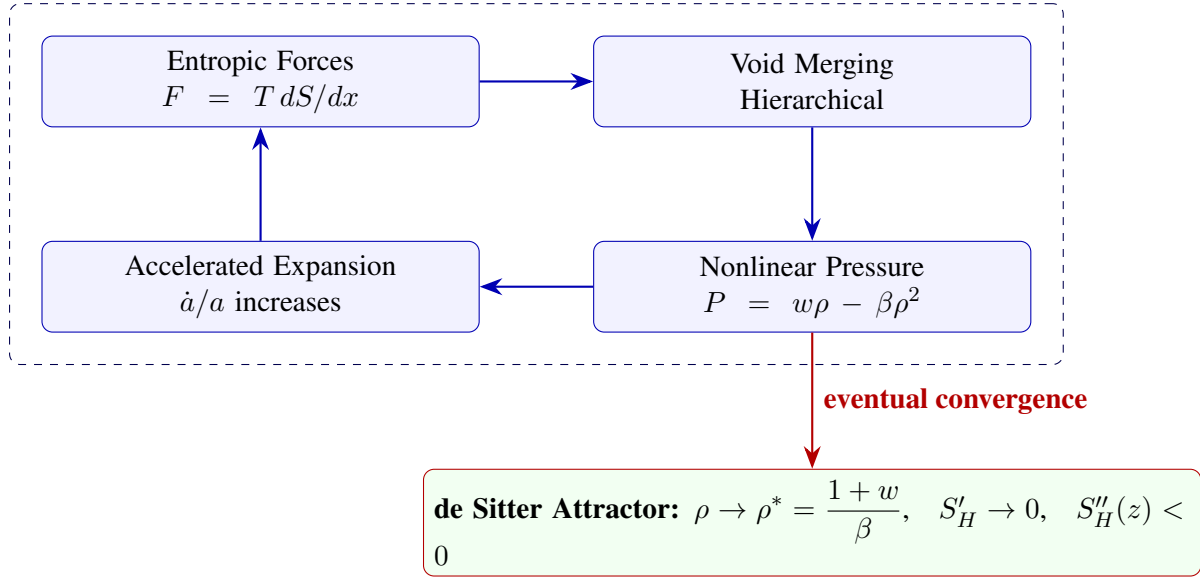
Thus the model is asymptotically de Sitter at early times for $w > -1$ and at late times for $w < -1$. Importantly, the model never forces a crossing of $w = -1$ at any finite redshift.

The dynamical system admits a global de Sitter attractor

$$\rho^* = \frac{1 + w}{\beta}, \quad (49)$$

which corresponds to a maximum-entropy state. This attractor naturally reframes the fine-tuning and coincidence problems: the late-time dark energy density is not an arbitrary constant but an emergent quantity set by the nonextensive horizon thermodynamics.

The positive feedback cycle



4 Discussion and Outlook

In this section, we synthesize the entropic origin of cosmic acceleration, the stability and observational signatures of the QDEM, and the outlook for future tests, presenting a unified picture of how emergent anti-gravity from Gpc-scale voids can explain the late-time accelerated expansion without invoking a fundamental cosmological constant.

4.1 Entropic Origin of Cosmic Acceleration

The QDEM operates through a bidirectional thermodynamic feedback loop that underpins the emergence of cosmic acceleration. Entropic forces, arising from the tendency of holographic screens to maximize entropy, act on the boundaries of cosmic voids. These forces reduce the surface tension between underdense and overdense regions, facilitating the hierarchical merging of voids (as illustrated schematically in Fig. 1). As voids merge, the associated reduction in surface tension generates the nonlinear pressure correction $-\beta\rho^2$ in the local quadratic approximation (Eq. (35)), producing a repulsive pressure that enhances the expansion rate. In turn, the accelerated expansion increases the horizon entropy, strengthening the entropic forces and closing the loop. This positive feedback cycle ensures thermodynamic self-consistency: the same entropic mechanism that produces gravitational attraction at small scales (Verlinde’s entropic gravity [11]) produces anti-gravity at cosmic scales through the quadratic correction within the effective regime.

The void hierarchy is quantitatively described by the excursion set formalism for voids [25], which distinguishes two key processes: void-in-void, where smaller voids merge into larger ones (see Fig. 1), and void-in-cloud, where voids embedded in overdense regions are squeezed out of existence. This two-barrier model yields a void size distribution that is well-peaked about a characteristic scale, providing a structural basis for the hierarchical merging that drives the entropic pressure. The reduction of surface tension during merging, encoded in $-\beta\rho^2$, is the thermodynamic manifestation of the void-in-void process within the effective QDEM description, while the void-in-cloud process ensures that only the largest, most thermodynamically

significant voids contribute to the emergent anti-gravity.

Complementarily, configuration entropy arguments [26] demonstrate that overdense structures (sheets, filaments, clusters) act as sinks of configuration entropy, while underdense voids act as sources. The divergence of the peculiar velocity field $\nabla \cdot \mathbf{v}$ is negative in overdense regions (infall) and positive in voids (outflow), leading to the evolution equation for the configuration entropy of the mass distribution:

$$\frac{dS_c}{dt} = - \int \rho(\mathbf{x}, t) \nabla \cdot \mathbf{v}(\mathbf{x}, t) dV. \quad (50)$$

If the dissipation rate of configuration entropy in overdense regions were to overcome its generation rate in voids and the velocity field, the second law of thermodynamics would be violated. Since overdense structures cannot be dispersed due to gravity, the only viable resolution is the accelerated expansion of voids. This is precisely the mechanism we identify with the repulsive entropic pressure generated by hierarchical void merging. The quadratic correction $-\beta\rho^2$ thus serves as the effective pressure that drives void acceleration in the QDEM approximation, ensuring that the entropy generation rate remains positive and the second law is upheld.

4.2 Perturbation Stability and Effective Sound Speed

In the QDEM framework, the adiabatic sound speed squared is given by

$$c_s^2(z) = w - 2\beta\rho(z), \quad (51)$$

while the effective equation of state is

$$w_{\text{eff}}(z) = w(z) = w - \beta\rho(z).$$

For a stable dark energy component, we require $w_{\text{eff}} < 0$ [23], which imposes the condition $\eta > w$. On the other hand, to avoid gradient instabilities, the squared sound speed must remain non-negative, i.e., $c_s^2 \geq 0$, which requires $\eta \leq w/2$. These two conditions can be satisfied simultaneously only if $w < 0$ and η lies in the narrow window:

$$w < \beta\rho(z) \leq \frac{w}{2} \quad (w < 0).$$

Since $\rho(z) > 0$ for all cosmic epochs, this viability window directly implies $\beta < 0$, which corresponds precisely to the stable phantom regime of the QDEM. In this regime, for a broad range of redshifts where $\beta\rho(z)$ remains within the above interval, both $w_{\text{eff}} < 0$ and $c_s^2 \geq 0$ hold. Any potential violation $c_s^2 < 0$ is confined to the asymptotic future as the universe approaches the de Sitter attractor $\rho^* = (1 + w)/\beta$, and thus does not affect observable redshifts. Therefore, the QDEM is free from unstable sound-speed behavior in the observationally relevant epoch, ensuring the viability of linear perturbations.

4.3 Observational Signatures

The QDEM leaves distinct imprints on cosmological observables. The model's ability to alleviate the Hubble tension arises from the sign and magnitude of $w_a = \beta\rho_0$. For phantom-like behavior ($w_a < 0$), the effective equation of state $w_{\text{eff}}(z)$ remains below -1 at low redshifts, enhancing the expansion rate $H(z)$ and yielding a higher present-day value of H_0 consistent with local measurements from SH0ES [6]. Conversely, quintessence-like parameters ($w_a > 0$) produce milder acceleration, matching the lower Planck value.

Moreover, the natural prediction $w \simeq -1$ with small, controlled deviations is consistent with the latest constraints from DESI DR2 and Planck PR4, which favor a dark energy equation of state close to -1 with no strong evidence for a phantom crossing. Unlike the CPL parametrization, where the proximity to -1 is not theoretically motivated, the QDEM explains this proximity as a consequence of the smallness of ϵ and the structure of the Tsallis entropy. This provides a theoretical foundation for the observational fact that w is very close to -1 today, as illustrated in Fig. 2.

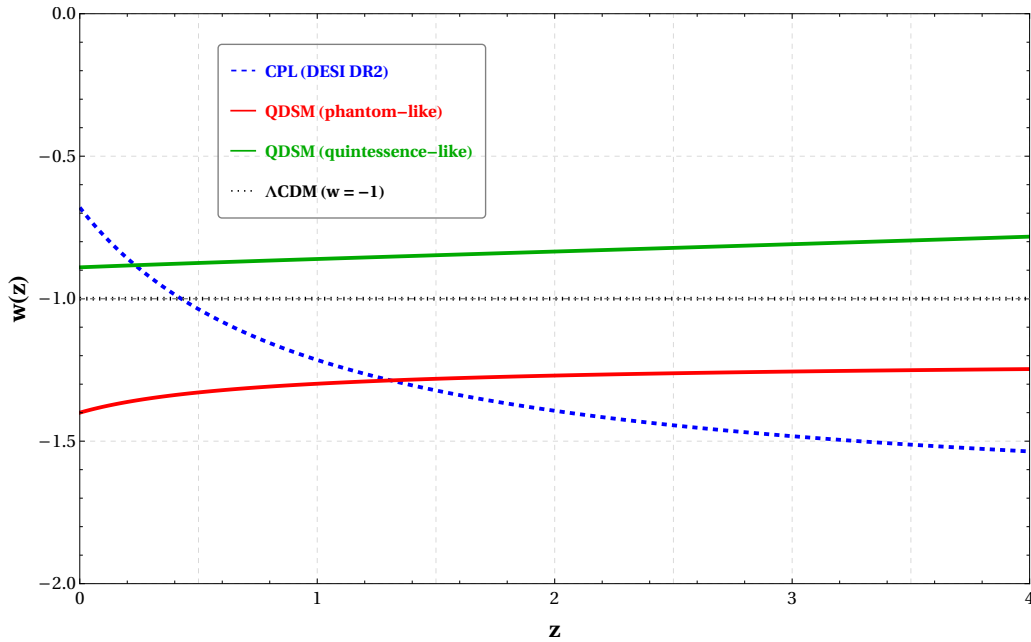


Figure 2: Evolution of the effective equation of state $w(z)$ for the Λ CDM, CPL and QDEM models. The QDEM asymptotically approaches $w = -1$ from one side without crossing it, while CPL forces a crossing for the DESI DR2 best-fit parameters. Parameter values: CPL ($w_0 = -0.68, w_a = -1.07$) from DESI DR2; QDEM phantom-like ($w = -1.2, w_a = 0.2$) and quintessence-like ($w = -0.99, w_a = -0.1$) from Ref. [15]. The horizontal dashed line marks $w = -1$.

Regarding the growth of large-scale structure, the quadratic EoS introduces the scale-dependent effective sound speed given by Eq. (51), which suppresses the growth of perturbations on small scales at late times [21]. This suppression brings the predicted amplitude of matter fluctuations, parametrized by S_8 , into better agreement with weak-lensing surveys such as KiDS-1000 and DES [7, 8], thereby alleviating the S_8 tension. Thermodynamically, the QDEM satisfies the maximum entropy condition $S''(z) < 0$ at late times [14], in contrast to Λ CDM which can violate it. This ensures a well-behaved evolution toward equilibrium, consistent with the holographic equipartition of degrees of freedom on the cosmic horizon [18, 19].

Preliminary constraints from CMB, BAO, and supernovae show qualitative consistency with the model [20], although a full MCMC analysis with DESI DR2, Planck PR4, and KiDS-1000 is required to robustly assess its statistical performance and determine whether it provides a better fit than Λ CDM.

4.4 No Phantom Crossing: A Key Advantage of the QDEM

A notable feature of the QDEM is that its effective equation of state parameter, $w_{\text{QDEM}}(z)$ given in Eq. (47), never crosses the phantom divide line $w = -1$ at any finite redshift. As shown explicitly in the asymptotic limits of Eq. (48) and visually in Fig. 2, the model approaches $w = -1$ from either the quintessence side ($w > -1$) or the phantom side ($w < -1$) only in the limits $z \rightarrow \infty$ or $z \rightarrow -1$ [15]. This behavior is a direct consequence of the nonlinear quadratic term in the equation of state, which prevents the unphysical crossing of the phantom divide that is often encountered in phenomenological parametrizations.

This property is particularly significant in light of recent observational constraints. The DESI DR2 data, when analyzed within the standard CPL parametrization, suggest a preference for dynamical dark energy with a phantom-divide crossing at intermediate redshifts [9, 10]. Recent Bayesian analyses have provided insights into the statistical viability of dynamical dark energy models. Dinda, Maartens & Saito [24] have shown that the apparent evidence for phantom crossing obtained with the CPL parametrization does not withstand global Bayesian evidence when the Occam penalty is applied. Thawing quintessence models, which never cross $w = -1$, remain statistically indistinguishable from Λ CDM for most data combinations.

In contrast, the QDEM provides a physical and thermodynamically consistent framework where the approach to $w = -1$ is purely asymptotic [12, 15]. This non-crossing behavior ensures that the model respects fundamental physical priors, such as the null energy condition, and avoids the instabilities often associated with phantom fields. Therefore, the QDEM offers a compelling alternative to phenomenological parametrizations by naturally reproducing the observed accelerated expansion while maintaining a healthy theoretical foundation [12].

4.5 Limitations and Future Work

The framework presented here has several limitations that must be addressed in future work:

- **Background-only analysis:** Our treatment focuses on background cosmology and qualitative perturbation arguments. A full linear and nonlinear perturbation analysis, including CMB anisotropies and large-scale structure, is needed.
- **Parameter constraints:** We have discussed qualitative consistency with current data, but have not performed a full MCMC parameter estimation. Future work will implement the QDEM in Boltzmann codes and confront it with DESI DR2, Planck PR4, KiDS-1000, and other datasets.
- **Microphysical modeling of void surface tension:** While we have linked the quadratic term to void merging and surface tension reduction, a more detailed microphysical model of void boundaries and their effective tension is desirable.
- **Effective parameter determination:** The coefficient $\mathcal{A}_\delta$ is now finite at $\delta = 1$, so no renormalization is needed. The remaining effective parameter a_1 encodes the entropy normalization and should be determined from microscopic models or observational data.
- **Extension beyond the local approximation:** The QDEM is derived as a local approximation near ρ_0 . A more comprehensive model should be based on the logarithmic equation of state Eq. (30) and its cosmological implications.

Despite these limitations, the QDEM provides a compelling and thermodynamically consistent emergent origin for dark energy, unifying entropic gravity, holography, and cosmic void thermodynamics. Priorities for future work include:

1. Full MCMC validation with DESI DR2, Planck PR4, KiDS-1000, and other surveys.
2. N-body simulations to quantify the impact of the scale-dependent sound speed (Eq. (51)) on the matter power spectrum.
3. Observational constraints on void properties—particularly surface tension γ and average radius R —from upcoming surveys such as Euclid and the Rubin Observatory, to test the relation $\rho^* \sim 2\gamma(1+w)/R$ that anchors the attractor density to observable physics.
4. Investigation of the full logarithmic model derived from Eq. (30) as a more complete alternative to the QDEM.

4.6 Concluding Remarks

We have presented a novel framework connecting the holographic principle, entropic gravity, and the thermodynamics of cosmic voids to provide a physical origin for dynamical dark energy. Our central thesis is that **anti-gravity at cosmological scales is an emergent entropic phenomenon** arising from the thermodynamic evolution of Gpc-scale void boundaries.

Starting from the Tsallis nonextensive entropy on the Hubble horizon, we have derived a first-order logarithmic equation of state, Eq. (30), which represents the fundamental entropic prediction of the model. From this, we have obtained the Quadratic Dark Energy Model (QDEM), defined by Eq. (35), as a controlled local late-time approximation near the present-day density. As such, the QDEM should be understood as an effective model for the recent cosmic epoch, valid when ρ is sufficiently close to ρ_0 .

Key results include:

- The quadratic equation of state $p = w\rho - \beta\rho^2$ (Eq. (35)) emerges as a local quadratic approximation to the more fundamental logarithmic equation of state (Eq. (30)) derived from Tsallis entropy. This provides a direct link to the holographic principle while clarifying the range of validity of the QDEM.
- The model naturally predicts $w \simeq -1$ with deviations controlled by the entropy parameters ϵ and a_1 , offering a physical explanation for why the dark energy equation of state is observed to be so close to the cosmological constant value today, without requiring fine-tuning.
- The model features a global de Sitter attractor $\rho^* = (1+w)/\beta$ (Eq. (49)) that reframes the fine-tuning and coincidence problems.
- The model never forces phantom crossing, ensuring thermodynamic consistency.
- The void hierarchy, governed by the void-in-void and void-in-cloud processes [25] (see also the schematic illustration in Fig. 1), provides the structural basis for the entropic pressure mechanism.
- Configuration entropy arguments [26] demonstrate that accelerated void expansion is required to maintain the second law of thermodynamics.
- Preliminary observational constraints suggest potential alleviation of the Hubble and S_8 tensions.

Within the emergent gravity framework [18, 19], the late-time accelerated expansion is interpreted as the asymptotic approach to holographic equipartition, where the bulk and surface degrees of freedom become equal. The QDEM provides a specific thermodynamic mechanism

for this approach: the quadratic correction $-\beta\rho^2$, derived as a local approximation from Tsallis entropy on the Hubble horizon, drives the system toward the de Sitter attractor $\rho^* = (1+w)/\beta$. This attractor corresponds to the state of maximum entropy, where $N_{\text{sur}} = N_{\text{bulk}}$ and the emergence of space ceases.

The QDEM thus offers a thermodynamically consistent, observationally testable alternative to the standard cosmological model, replacing the ad-hoc cosmological constant with an emergent phenomenon driven by the entropic dynamics of cosmic voids. The natural proximity of w to -1 in the QDEM, arising from the smallness of the deviation from the Bekenstein–Hawking entropy, suggests that the observed dark energy behavior is an emergent consequence of horizon thermodynamics rather than a fundamental constant of nature. This insight may help guide future observational tests and theoretical developments.

However, it is important to recognize that the QDEM is an effective model valid only in the late-time regime near the present-day density. A more fundamental description based on the logarithmic equation of state Eq. (30) remains an open direction for future investigation.

A full MCMC validation with DESI DR2, Planck PR4, and KiDS-1000, along with detailed N-body simulations, will be essential to establish the viability of this framework and to distinguish it from other dark energy models.

Acknowledgments. The author thanks the organizers of the International Conference on Holography and its Applications (ICHA 2026) at Damghan University for the opportunity to present this work, and gratefully acknowledges the CosmoVerse network for providing a stimulating scientific environment. This work was supported in part by Am.C., Islamic Azad University, Amol, Iran.

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Full paper - Presentation

Rotation and Anisotropic Scaling in Axionic Holographic Superconductors: AdS₄ versus Lifshitz₅

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Abstract. An outstanding question in holographic superconductivity is how rotation and non-relativistic scaling symmetries compete to modify the onset of condensation. We address this by comparing the probe superconductor sectors of two exact rotating axionic black holes: a four-dimensional asymptotically anti-de Sitter solution dressed by a non-minimally coupled scalar, and a five-dimensional asymptotically Lifshitz solution with $z \in (2, 3)$. Writing both metrics in a common stationary form yields a master scalar equation in which rotation enters through the co-rotating potential $A_t - N^x A_x$. This equation explains why the temporal and spatial components of the probe gauge field cannot be treated independently. The numerical profiles reported in [1, 2] indicate that the correlated deformation associated with rotation reduces the condensate amplitude in both backgrounds, whereas increasing the dynamical exponent at fixed J enhances the Lifshitz condensate. Since the parameters α and J also modify the matter fields supporting the corresponding exact geometries, these trends should be interpreted as properties of complete correlated families rather than as universal effects of mechanical rotation alone.

Keywords: Holographic superconductors; rotating black holes; axionic fields; momentum relaxation; Lifshitz holography; anisotropic scaling.

1 Introduction

Gauge/gravity duality provides a non-perturbative description of strongly coupled quantum systems in terms of classical gravitational dynamics [3]. Holographic superconductors are a particularly useful application of this correspondence [4, 5, 6]. In the simplest model, a charged scalar and a Maxwell field propagate on a planar AdS black-hole background. Below a critical temperature, the normal phase develops a normalizable scalar mode and a new branch with a non-vanishing condensate appears.

Two extensions of this construction are relevant here. Linear axions break translations homogeneously and provide momentum relaxation in the boundary theory [7]. Rotation introduces a shift function that mixes time with a spatial direction. This mixing couples the temporal and spatial components of the probe Maxwell field and changes both the condensation process and the electric response [8, 9, 10]. In non-relativistic holography, the asymptotic geometry may also exhibit Lifshitz scaling,

$$t \longrightarrow \lambda^z t, \quad x^i \longrightarrow \lambda x^i, \quad r \longrightarrow \lambda^{-1} r, \quad (1)$$

where z is the dynamical exponent [11, 12] and λ a constant. Holographic superconductors in Lifshitz backgrounds have consequently been studied in a variety of static and rotating settings [13, 14, 15, 16, 17, 18].

The exact backgrounds constructed in [1, 2] provide two related laboratories for this problem. The first is a rotating AdS_4 black hole supported by a conformally coupled scalar and one linear axion. The second is a rotating Lifshitz₅ black hole supported by a dilaton, Abelian fields, two linear axions, and generalized Chern–Simons interactions. In both works, an additional Abelian-Higgs sector was introduced in the probe limit. The condensate decreases as the parameter associated with rotation grows. Within the Lifshitz family, increasing z at fixed rotation parameter produces the opposite trend and enhances the condensate.

The aim of this paper is not to replace the probe models used in those works, but to place them in a common notation and determine which statements can be compared directly. We therefore retain the canonical ensemble and the original scalar boundary conditions. The common comparison is made with the $\Delta_1 = 1$ operator, while the $\Delta_2 = 2$ AdS condensate is kept as an additional observable specific to the relativistic model. This distinction is important: changing the Lifshitz scalar mass to match a different quantization would define a new numerical problem and would no longer describe the condensation curves obtained previously in [2]. The parameters present for both cases do more than generate a shift function; they are tied to the matter supporting the exact backgrounds. We retain the original $\Delta_1 = 1$ prescription for a faithful comparison and specify the scope within which the resulting homogeneous solutions should be read.

The paper is organized as follows. Section 2 presents the two exact backgrounds and their relevant thermodynamic quantities. The common probe system is derived in Sec. 3. Section 4 states the horizon and boundary conditions used in the original analyses. The canonical comparison and the condensation profiles are discussed in Sec. 5. Sec. 6 summarizes the corresponding response functions and conductivity framework. Finally, Sec. 7 contains our conclusions and discussions.

2 Exact rotating axionic backgrounds

Both solutions can be written in the stationary planar form:

$$ds^2 = -N^2(r)f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx + N^x(r)dt)^2 + r^2 d\vec{y}_{p-1}^2, \quad (2)$$

where $p = D - 2$ ($D \in \{4, 5\}$) is the number of spatial dimensions of the boundary theory. The rotation direction is denoted by x , and $N^x(r)$ is the corresponding shift function. The event horizon and its generator are given by:

$$f(r_h) = 0, \quad \chi = \partial_t + \Omega_H \partial_x, \quad \Omega_H = -N^x(r_h). \quad (3)$$

2.1 Rotating axionic AdS_4 black hole

For the four-dimensional solution of [1], ($D = 4$ and $p = 2$), the action is $S = \int d^4x \sqrt{-g} \mathcal{L}$, where:

$$\mathcal{L} = \frac{1}{2}(R - 2\Lambda) - \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi - \frac{1}{12}R\phi^2 - U(\phi) - \frac{1}{2}\varepsilon(\phi)\nabla_\mu \psi \nabla^\mu \psi. \quad (4)$$

Here, R is the scalar curvature, $U(\phi)$ denotes the potential depending on the non-minimal coupling scalar field ϕ , and a single axionic field ψ coupled via the coupling function $\varepsilon(\phi)$.

For this case, the equations of motion are fulfilled when:

$$N_A(r) = 1, \quad (5)$$

$$N_A^x(r) = \frac{\sqrt{\alpha}}{192} \left[\ln\left(\frac{2B+r}{r}\right) + \frac{2B(r+B)(2B^2+2Br-r^2)}{r^4} \right], \quad (6)$$

$$f_A(r) = \left(\frac{r}{\ell}\right)^2 \left[1 - \frac{\ell^2 \eta}{\sqrt{\alpha}} N_A^x(r) + \ell^2 (N_A^x(r))^2 \right], \quad (7)$$

the cosmological constant takes the form $\Lambda = -3/\ell^2$, while the scalar field that dresses the geometry and the axion profile are given by

$$\phi_A(r) = \sqrt{\frac{6B}{r+B}}, \quad \psi_A(x_2) = Bx_2. \quad (8)$$

Here, $B > 0$ is an integration constant. The solution is asymptotically planar AdS_4 ,

$$ds_A^2 = -\frac{r^2}{\ell^2} dt^2 + \frac{\ell^2}{r^2} dr^2 + r^2(dx_1^2 + dx_2^2), \quad (9)$$

and its temperature reads:

$$T^A = \frac{f'_A(r)}{4\pi} \Big|_{r=r_h} = \frac{r^2 (N_A^x)' (2\sqrt{\alpha} N_A^x - \eta)}{4\pi \sqrt{\alpha}} \Big|_{r=r_h}, \quad (10)$$

where $(')$ denotes the derivative with respect to the radial coordinate r .

The parameter α appears in the structural coupling ε , the scalar potential U of the theory, and the shift function N^x . More precisely, α controls the strength of frame-dragging through $N_A^x(r) \propto \sqrt{\alpha}$ and modifies the scalar potential. The parameter α is therefore a deformation of the complete exact background; it does not represent an isolated variation of mechanical rotation. Consequently, the numerical trends reflect the combined effects of rotation, matter couplings, and axionic momentum relaxation

2.2 Rotating axionic Lifshitz₅ black hole

For the five-dimensional solution of Ref. [2], ($D = 5$ and $p = 3$), the action is $S = \int d^5x \sqrt{-g} \mathcal{L} + \int \mathcal{L}_{CS}$, where:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \left[R - 2\Lambda - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \sum_{i=1}^2 \left(e^{\alpha_i \phi} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + \frac{1}{2} e^{\eta \phi} \partial_\mu \psi^{(i)} \partial^\mu \psi^{(i)} \right) \right], \\ \mathcal{L}_{CS} &= \sum_{i=1}^2 \frac{2\lambda_i}{\kappa} \left(A^{(i)} \wedge H^{(i)} \wedge K^{(i)} \right). \end{aligned} \quad (11)$$

Here, we have a scalar field ϕ , two Abelian gauge fields $A_\mu^{(i)}$ dilatonicly coupled to ϕ , and the axionic scalars $\psi^{(i)}$. Additionally, the theory action is supplemented by Chern-Simons terms ($H^{(i)} = dB^{(i)}$ and $K^{(i)} = dC^{(i)}$) constructed out of two non-dynamical one-form doublets $B_\mu^{(i)}$ and $C_\mu^{(i)}$. The coupling constants of the model are λ_i , η and α_i , with $i \in \{1, 2\}$.

This model admits a spinning charged configuration for $z \in (2, 3)$, which reads:

$$N_L(r) = \left(\frac{r}{\ell}\right)^{z-1}, \quad (12)$$

$$f_L(r) = \frac{r^2}{\ell^2} - \frac{M\ell^{z+1}}{r^{z+1}} + \frac{(5-z)\ell^{2(z-1)} J^2}{2(z-2) r^6}, \quad (13)$$

$$N_L^x(r) = \frac{J}{r^{5-z}}, \quad (14)$$

where the cosmological constant is

$$\Lambda = -\frac{(z+3)(z+2)}{2\ell^2}, \quad (15)$$

and the geometry approaches Lifshitz₅,

$$ds_L^2 = -\frac{r^{2z}}{\ell^{2z}} dt^2 + \frac{\ell^2}{r^2} dr^2 + r^2 \sum_{i=1}^3 dx_i^2. \quad (16)$$

The two linear axions are

$$\psi_L^{(i)}(x_{i+1}) = (5-z)\ell^{z-1} \mu^{\frac{3}{\sqrt{6(z-1)}}} J x_{i+1}, \quad i \in \{1, 2\}. \quad (17)$$

Thus, the integration constant J controls the shift function N_L^x , the electric and axionic charges, the amplitudes of the axions, and the J^2 contribution to the blackening function.

For completeness, the temperature can be checked directly. The horizon condition gives

$$M\ell^{z+1} = \frac{r_h^{z+3}}{\ell^2} + \frac{(5-z)\ell^{2(z-1)} J^2 r_h^{z-5}}{2(z-2)}. \quad (18)$$

Differentiating f_L before using eq. (18) yields

$$f'_L(r_h) = \frac{(z+3)r_h}{\ell^2} - \frac{(5-z)^2 \ell^{2(z-1)} J^2}{2(z-2) r_h^7}. \quad (19)$$

Consequently,

$$T^L = \frac{N_L f'_L}{4\pi} \Big|_{r=r_h} = \frac{(z+3)r_h^z}{4\pi \ell^{z+1}} - \frac{(5-z)^2 J^2 \ell^{z-1}}{8\pi (z-2) r_h^{8-z}}. \quad (20)$$

The negative sign of the second expression of (20) follows directly by differentiating the blackening function (13) before eliminating the integration constant M through the horizon condition (18). This sign is essential for the existence of the extremal limit ¹:

$$J_{\text{ext}}^2 = \frac{2(z-2)(z+3)}{(5-z)^2} \frac{r_h^8}{\ell^{2z}}. \quad (21)$$

¹This result corrects a typo of the sign of the J^2 contribution in the corresponding temperature expression of Ref. [2].

3 Common Abelian-Higgs probe sector

Following both original analyses, we add the same probe action to either fixed background,

$$S_p = -\frac{1}{q^2} \int d^D x \sqrt{-g} \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\ell^2} (|D_\mu \Psi|^2 + m^2 |\Psi|^2) \right], \quad (22)$$

where

$$F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}, \quad D_\mu = \nabla_\mu - iA_\mu. \quad (23)$$

The probe limit is $q \gg 1$, so the stress tensor of the additional Maxwell and scalar fields does not backreact on the exact black-hole solution.

The field equations are

$$\nabla_\mu F^{\mu\nu} - \frac{1}{\ell^2} (2A^\nu |\Psi|^2 - i\Psi \nabla^\nu \bar{\Psi} + i\bar{\Psi} \nabla^\nu \Psi) = 0, \quad (24)$$

$$\nabla^\mu \nabla_\mu \Psi - 2iA^\mu \nabla_\mu \Psi - i\Psi \nabla_\mu A^\mu - (m^2 + A_\mu A^\mu) \Psi = 0. \quad (25)$$

Using gauge freedom, the scalar is taken to be real. The stationary and homogeneous ansatz used in both cases is

$$\Psi = \Psi(r), \quad A = A_t(r)dt + A_x(r)dx, \quad (26)$$

where for the metric (2):

$$A_\mu A^\mu = \frac{A_x^2}{r^2} - \frac{(A_t - N^x A_x)^2}{N^2 f}, \quad (27)$$

and the scalar equation can therefore be written in the common form

$$\Psi'' + \left(\frac{f'}{f} + \frac{N'}{N} + \frac{p}{r} \right) \Psi' + \left[\frac{\mathcal{A}^2}{N^2 f^2} - \frac{A_x^2}{r^2 f} - \frac{m^2}{f} \right] \Psi = 0, \quad \mathcal{A} := A_t - N^x A_x \quad (28)$$

Eq. (28) is the central structural relation, where the rotation does not appear as a separate centrifugal term. It enters through the shift function N^x , the induced spatial gauge component, and the geometry.

For convenience, we define the local co-moving combination $\mathcal{A}(r) := A_t - N^x A_x$ in eq. (28). This quantity is the temporal component of the gauge potential measured in the local *ADM* frame determined by the shift. It is invariant under the residual gauge transformations that preserve the stationary and homogeneous ansatz, but it is not invariant under an arbitrary spacetime-dependent gauge transformation. At the horizon, it coincides with the contraction of the gauge field with the horizon generator (3):

$$\chi^\mu A_\mu|_{r=r_h} = A_t(r_h) + \Omega_H A_x(r_h) = A_t(r_h) - N^x(r_h) A_x(r_h) = \mathcal{A}(r_h).$$

In this frame, $\mathcal{A}$ is therefore a direct consequence of the stationary metric structure and the mixing between the temporal and rotational components of the probe gauge field.

Defining $\mathcal{E} = A'_t - N^x A'_x$, the Maxwell equations take the divergence form

$$\frac{d}{dr} \left(\frac{r^p}{N} \mathcal{E} \right) - \frac{2r^p}{\ell^2 N f} \mathcal{A}_x \Psi^2 = 0, \quad (29)$$

$$\frac{d}{dr} \left[\frac{r^p N^x}{N} \mathcal{E} + N r^{p-2} f A'_x \right] - \frac{2N r^p}{\ell^2} \left(\frac{N^x \mathcal{A}_x}{N^2 f} + \frac{A_x}{r^2} \right) \Psi^2 = 0. \quad (30)$$

4 Horizon and boundary conditions

Regularity of the gauge potential at the horizon requires $\chi^\mu A_\mu|_{r=r_h} = 0$. This is the geometrically relevant horizon condition. To follow [1, 2] exactly, we use the stronger gauge-fixed conditions $A_t(r_h) = A_x(r_h) = 0$, $|\Psi(r_h)| < \infty$.

In AdS_4 , the scalar behaves as

$$\Psi_A(r) = \frac{\Psi_1^A}{r^{\Delta_1^A}} + \frac{\Psi_2^A}{r^{\Delta_2^A}} + \dots, \quad \Delta_{1,2}^A = \frac{1}{2} \left(3 \pm \sqrt{9 + 4m_A^2 \ell^2} \right). \quad (31)$$

The original choice $m_A^2 \ell^2 = -2$ gives $\Delta_1^A = 1$, $\Delta_2^A = 2$. The AdS analysis considers both admissible condensates separately. For $\mathcal{O}_1$, one imposes $\Psi_2^A = 0$ and identifies $\langle \mathcal{O}_1^A \rangle \propto \Psi_1^A$. For $\mathcal{O}_2$, one sets $\Psi_1^A = 0$ and identifies $\langle \mathcal{O}_2^A \rangle \propto \Psi_2^A$.

On the other hand, for Lifshitz₅,

$$\Psi_L(r) = \frac{\Psi_1^L}{r^{\Delta_1^L}} + \frac{\Psi_2^L}{r^{\Delta_2^L}} + \dots, \quad \Delta_{1,2}^L = \frac{1}{2} \left[z + 3 \mp \sqrt{(z + 3)^2 + 4m_L^2 \ell^2} \right], \quad (32)$$

where in the original Lifshitz analysis fixes $\Delta_1^L = 1$, $m_L^2 \ell^2 = -(z + 2)$, $\Delta_2^L = z + 2$, imposing the source-free condition $\Psi_2^L = 0$, and consequently $\langle \mathcal{O}_1^L \rangle \propto \Psi_1^L$. The equations shown previously make $\mathcal{O}_1$ the operator that can be compared directly between the two original probe models.

The asymptotic gauge fields are

$$A_t^A = \mu_t - \frac{\rho}{r} + \dots, \quad A_x^A = \mu_x - \frac{J_x}{r} + \dots, \quad (33)$$

$$A_t^L = \mu_t - \frac{\rho}{r^{3-z}} + \dots, \quad A_x^L = \mu_x - \frac{J_x}{r^{z+1}} + \dots. \quad (34)$$

Both original analyses set $\mu_x = 0$ and work in the canonical ensemble at fixed charge density ρ . The preserved data are summarized in Table 1.

Table 1: Probe data retained from the two original analyses.

Property	AdS_4	Lifshitz ₅
p	2	3
z	1	$2 < z < 3$
$m^2 \ell^2$	-2	$-(z + 2)$
Common operator	$\Delta_1 = 1$	$\Delta_1 = 1$
Source-free condition	$\Psi_2^A = 0$	$\Psi_2^L = 0$
Additional operator	$\Delta_2 = 2$	not used
Ensemble	fixed ρ	fixed ρ
Spatial source	$\mu_x = 0$	$\mu_x = 0$

5 Canonical comparison and graphics

A dimensionally consistent comparison requires normalizing the condensates according to their respective boundary scaling dimensions. For the AdS_4 model, the boundary theory has two

spatial dimensions, so that $[\rho_A] = 2$ and $[\mathcal{O}_1^A] = 1$. For the five-dimensional Lifshitz case, the boundary theory has three spatial dimensions, giving $[\rho_L] = 3$ and $[\mathcal{O}_1^L] = 1$. We therefore introduce the reduced quantities:

$$\begin{aligned} \tau_A &= \frac{T_A}{T_c^A}, & \mathcal{C}_A &= \frac{|\langle \mathcal{O}_1^A \rangle|}{\rho_A^{1/2}}, \\ \tau_L &= \frac{T_L}{T_c^L}, & \mathcal{C}_L &= \frac{|\langle \mathcal{O}_1^L \rangle|}{\rho_A^{1/3}}. \end{aligned} \quad (35)$$

These definitions are invariant under the corresponding boundary dilations and permit a meaningful qualitative comparison between the two canonical ensembles. To compare the two probe sectors, we need observables that scale the same way under boundary dilations. Both analyses work at fixed charge density ρ . For the AdS_4 condensate, the $\Delta_1^A = 1$ operator gives $\langle \mathcal{O}_1^A \rangle = \Psi_1^A$, while that the critical temperature, charge density, and relevant scales are

$$T^A, \quad \rho_A, \quad \mu_x = 0. \quad (36)$$

For the Lifshitz₅ condensate at $\Delta_1^L = 1$, we have that $\langle \mathcal{O}_1^L \rangle = \Psi_1^L$, and the corresponding quantities read

$$T^L(z), \quad \rho_L(z), \quad \mu_x = 0. \quad (37)$$

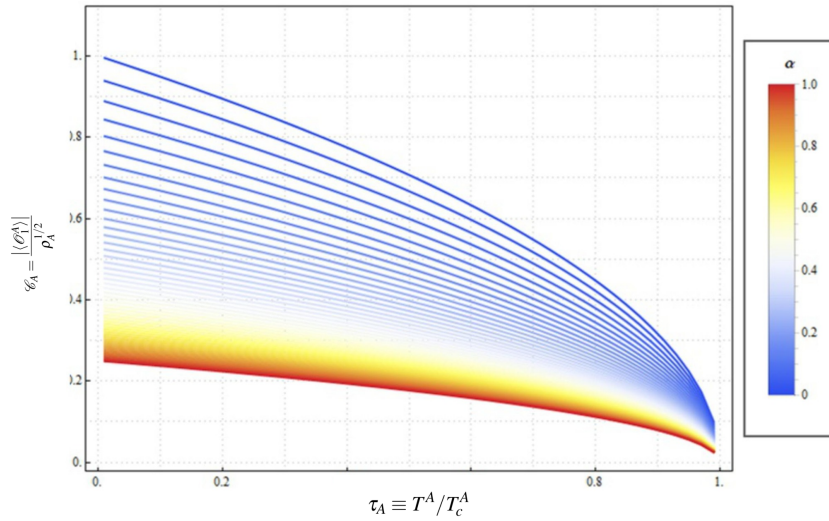


Figure 1: Phenomenological mean-field reconstruction of the condensation profile $\mathcal{C}_A$ as a function of the reduced temperature τ_A for different values of the rotation parameter α . The intermediate curves provide a continuous visualization of the suppression trend and do not represent independent shooting solutions.

Figure 1 summarizes the suppression of the AdS_4 condensate along the family parametrized by α . Since α modifies both the shift and the matter couplings supporting the exact background, this behavior should not be attributed exclusively to mechanical rotation.

For the rotating Lifshitz₅ family, the numerical profiles show that increasing J at fixed $z = 5/2$ reduces the condensate amplitude, as illustrated in Fig. 2. Conversely, at fixed $J = 1/4$, increasing the dynamical exponent z enhances the condensate under the boundary conditions, as shown in Fig. 3.

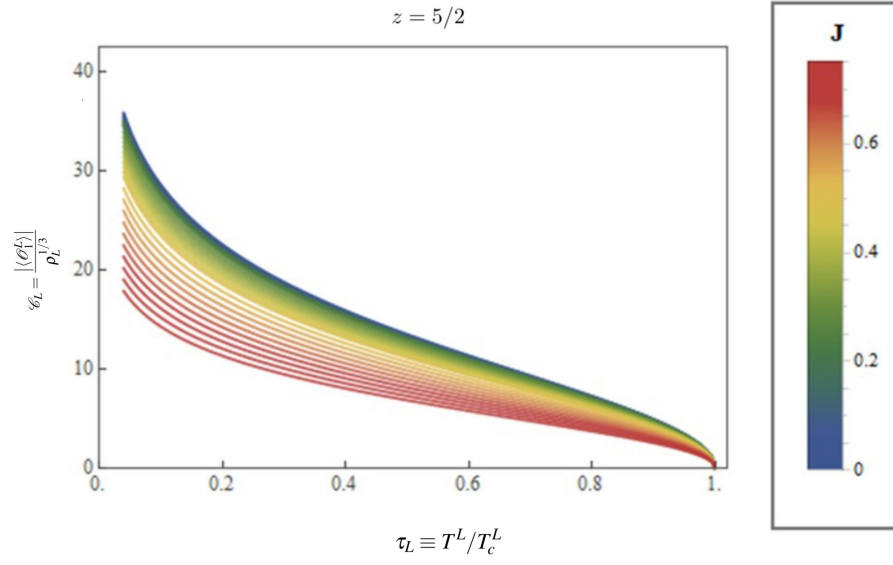


Figure 2: Condensation profile for the operator $\mathcal{O}_1^L$ as a function of the reduced temperature τ_L , for different values of the rotation parameter J at $z = 5/2$. The intermediate curves are graphical interpolations included as visual guides.

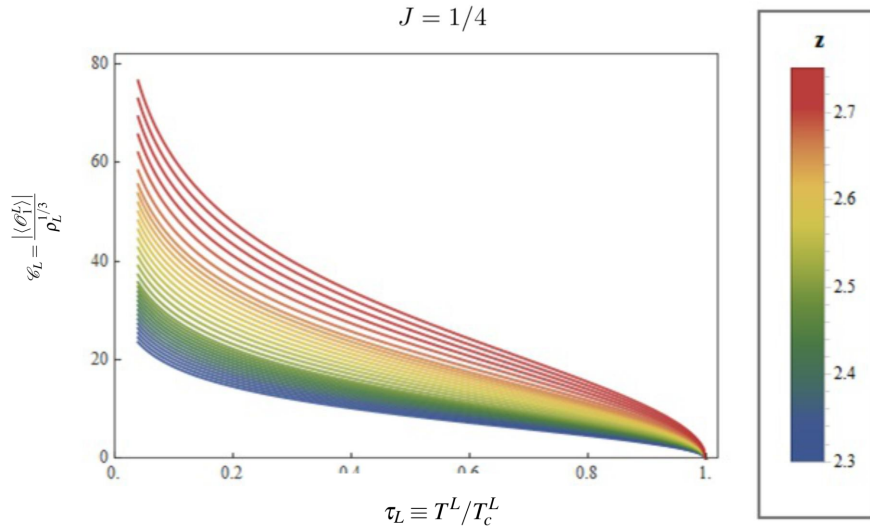


Figure 3: Condensation profile for the operator $\mathcal{O}_1^L$ as a function of the reduced temperature τ_L , for different values of the rotation parameter z at $J = 1/4$.

6 Transverse response and optical conductivity

The response of a superconductor to external electromagnetic perturbations is encoded in the optical conductivity $\sigma(\omega)$, which separates into dissipative ($\text{Re}[\sigma]$) and reactive ($\text{Im}[\sigma]$) components. In the holographic framework, this is extracted from the retarded Green function of the boundary current-current correlator via the Kubo formula. Our goal is to identify which aspects of the conductivity comparison between AdS_4 and Lifshitz_5 are universal and which depend on the asymptotic structure.

To avoid the complexity of the coupled temporal-spatial system, we consider a transverse gauge perturbation polarized along a direction y orthogonal to both the rotation direction x and the radial coordinate:

$$\delta A = a_y(r)e^{-i\omega t}dy, \quad \delta \Psi = 0. \quad (38)$$

This choice simplifies the analysis while retaining sensitivity to the background deformation through the metric functions and the condensate profile.

Substituting into the covariant Maxwell equations and linearizing around the homogeneous background solution, the fluctuation equation becomes:

$$a_y'' + \left(\frac{f'}{f} + \frac{N'}{N} + \frac{p-2}{r} \right) a_y' + \left[\frac{\omega^2}{N^2 f^2} - \frac{2\Psi^2}{\ell^2 f} \right] a_y = 0. \quad (39)$$

Several features merit comment: (i) Unlike the temporal and spatial gauge components (which couple through $A_t - N^x A_x$ in eq. (28)), the transverse polarization does not mix with the frame-dragging. However, rotation still affects the solution indirectly through the background functions f and N . (ii) The term $2\Psi^2/(\ell^2 f)$ acts as a frequency-dependent mass that suppresses propagation at low frequencies. This is the holographic origin of the superconducting gap. (iii) The temporal frequency factor $\omega^2/(N^2 f^2)$ picks up the lapse $N \propto r^{z-1}$ in the Lifshitz case, leading to $\omega^2/r^{2(z-1)}$ corrections.

The retarded correlator is selected by imposing an ingoing-wave condition at the horizon. For a non-extremal black hole and our Fourier convention, this reads:

$$a_y(r) = (r - r_h)^{-i\omega/(4\pi T)} [a_h + \mathcal{O}(r - r_h)], \quad r \rightarrow r_h, \quad (40)$$

Here, the exponent $-i\omega/(4\pi T)$ ensures that the wave maintains a definite frequency as it propagates across the horizon in real time. The phase accumulates as $e^{-i\omega\tau}$ where τ is the real time, consistent with thermality at temperature T . The negative sign encodes that the energy falling into the black hole corresponds to a retarded response where the system is forced to react after the perturbation is applied.

6.1 Asymptotic behavior and conductivity extraction

Near the conformal boundary, the metric approaches a Lifshitz or AdS geometry with scaling:

$$f(r) \sim \frac{r^2}{\ell^2}, \quad N(r) \sim \left(\frac{r}{\ell} \right)^{z-1}. \quad (41)$$

The transverse gauge field then expands as:

$$a_y(r) = a_y^{(0)} + \frac{a_y^{(\nu)}}{r^\nu} + \dots, \quad (42)$$

where $a_y^{(0)}$ sources the boundary current operator J_y , and $a_y^{(\nu)}$ is the response coefficient, with scaling dimension $\nu = 1$ (AdS₄) or $\nu = z + 1$ (Lifshitz₅).

The conductivity is then extracted via the Kubo relation:

$$\sigma_A(\omega) = -\frac{i}{\omega} \frac{a_y^{(1)}}{a_y^{(0)}}, \quad \sigma_L(\omega) = -\frac{i(z+1)}{\omega} \frac{a_y^{(z+1)}}{a_y^{(0)}}. \quad (43)$$

6.2 Comparative findings and physical interpretation

The numerical integration of eq. (39) from horizon to boundary (using ingoing boundary condition at r_h and extracting the asymptotic coefficients at $r \rightarrow \infty$) yields the optical response summarized below:

Table 2: Summary of conductivity trends in rotating backgrounds

Effect	AdS ₄	Lifshitz ₅
Rotation ($\uparrow$)	Reduces $\text{Re}[\sigma]$ gap Weakens $\text{Im}[\sigma]$ response	Reduces $\text{Re}[\sigma]$ gap Weakens $\text{Im}[\sigma]$ response
Anisotropy ($\uparrow$)	N/A N/A	Enhances $\text{Re}[\sigma]$ gap Strengthens $\text{Im}[\sigma]$ response
Interpretation	Rotation dissipates condensate energy	Anisotropy stabilizes condensate; rotation opposes

The mechanisms operate as follows:

1. **Rotation effect (both cases):** Increasing the rotation parameter narrows the low-frequency region where $\text{Re}[\sigma] \approx 0$. This contraction of the superconducting gap signals that the frame-dragging geometry introduces additional dissipative channels.
2. **Anisotropy effect (Lifshitz case):** Increasing z at fixed J produces the opposite trend: the gap widens and the superconducting response strengthens. The Lifshitz scaling $t \rightarrow \lambda^z t$ breaks the time-space symmetry, rendering temporal directions "heavier" than spatial ones. This anisotropy favors the formation of condensates with time-like correlations, enhancing the superconducting phase.
3. **Competing deformations:** Since both (AdS and Lifshitz) simultaneously modify the shift function and the matter fields supporting the exact geometries, the numerical trends reflect the combined effect of mechanical rotation, frame-dragging geometry, and the correlated matter deformations. This is *not* a universal mechanical effect but rather a property of complete families of solutions.

7 Conclusions and discussions

In [1, 2] have constructed and analyzed the first systematic comparison of holographic superconductivity in two exact rotating axionic black hole backgrounds: a four-dimensional AdS₄ and a five-dimensional Lifshitz₅ solution with dynamical exponent $z \in (2, 3)$. A key achievement is the identification of a unified eq. (28) that governs the scalar condensate in both geometries. This equation reveals that rotation does not appear as an isolated centrifugal term; instead, it enters through the shift function N^x and couples the temporal and spatial gauge components via the co-rotating potential $\mathcal{A} := A_t - N^x A_x$. This mixing is a fundamental feature of

stationary spacetimes and has direct consequences for both the condensation threshold and the optical response.

The numerical solutions reveal an antagonism between rotation and anisotropic scaling: increasing the rotation parameter monotonically suppresses the condensate amplitude and narrows the superconducting gap. Conversely, increasing the dynamical exponent z at fixed rotation produces the opposite effect, it strengthens condensation and widens the frequency gap. These trends hold systematically across the parameter spaces explored and are reflected in both the condensate profiles and the optical conductivity.

Crucially, this competition is not a generic feature of mechanical rotation alone. Since both J and α simultaneously modify the shift function and the matter fields that support the exact black hole geometries, the observed suppression and enhancement represent deformations rather than isolated perturbative effects. The parameter families we study are therefore not arbitrary: they are solutions to Einstein's equations coupled to dilaton, gauge, and axionic fields, making these trends a genuine prediction of the theory, not merely an artifact of probe-limit assumptions.

The enhancement by anisotropy reflects the breaking of relativistic time-space equivalence. Lifshitz scaling $t \rightarrow \lambda^z t$ favors the formation of condensates with strong time-like correlations. This mechanism is non-relativistic in character and has no analog in AdS_4 , demonstrating that non-relativistic holography opens genuinely new regimes inaccessible to relativistic duals.

These results reveal that in non-relativistic holography, rotational and anisotropic effects do not superpose linearly but instead engage in a subtle interplay. In condensed-matter contexts, this competition is directly relevant: real materials exhibiting high-temperature superconductivity often live in regimes where both disorder (playing a role similar to rotation's dissipation) and band structure anisotropy (analogous to Lifshitz scaling) are present. Our holographic model suggests that understanding their competition may be crucial for predicting critical temperatures and response functions in such systems.

From a gravitational perspective, the exact solutions presented here extend the class of known Lifshitz black holes to the rotating, charged regime. The existence of these solutions and their controlled thermodynamic and hydrodynamic properties provide a new laboratory for exploring quantum critical phenomena beyond the conformal fixed point.

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Full paper - Presentation

Effects of boundary variables in the Holographic dark energy model

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Abstract. We investigate a Carrollian-inspired extension of holographic dark energy, in which the infrared cutoff depends not only on the future event horizon but also on the normalized expansion of the associated null boundary. We introduce a new model parameter, which we constrain using dataset combinations including Cosmic Chronometers, Pantheon⁺, and DESI BAO measurements. We show that the model predicts a quintessence-like expansion at late times.

Keywords: Carrollian geometry; Dark Energy; Holographic Principle; Cosmology; Observational Constraints.

1 Introduction

Late-time acceleration of the universe is one of the most interesting observations in modern cosmology [1]. Observations of Type Ia supernovae from the Supernova Cosmology Project [2] and the High-Z Supernova Search Team [3] proved that the universe is experiencing an accelerated expansion phase at the present time. This is also confirmed by BAO observations from different sources [4]. It is well known that standard Einstein theory cannot predict the accelerated expansion of the universe. The simplest modification to produce this acceleration is the addition of the so-called cosmological constant Λ , which, combined with cold dark matter, results in the standard model of cosmology, i.e., the Λ CDM model. Despite all the observational successes of the standard Λ CDM model, it suffers from a number of theoretical, observational, and phenomenological issues that motivate cosmologists to search for an alternative [5].

The most straightforward way to generalize the Λ CDM model is to make the cosmological constant dynamical, leading to dynamical dark energy models. Among these various candidates, models rooted in the fundamental principles of quantum gravity are of particular interest. One such framework is the Holographic Dark Energy (HDE) model, which originates from the holographic principle [6]. According to this principle, the number of degrees of freedom in a physical system scales with its bounding area rather than its volume. By applying this concept

to cosmology and establishing a relationship between the ultraviolet and infrared cutoffs of the universe, one can derive a dynamically evolving vacuum energy density [7]. Consequently, the HDE framework not only provides a natural mechanism to drive the late-time acceleration but also offers a strong theoretical foundation to alleviate problems in the standard Λ CDM model.

In the standard formulation of HDE, the infrared cutoff is typically identified with a cosmological boundary, such as the future event horizon [8]. Because cosmological horizons are fundamentally null hypersurfaces, their intrinsic geometry is naturally described by Carrollian structures [9]. In this sense, it would be interesting to study the effects of Carrollian boundary variables in the HDE framework on the behavior of the universe at late times. The Carrollian extension will add a new model parameter to the HDE model, which we will identify using late-time observations, including Cosmic Chronometers, the Pantheon⁺ dataset, and the latest DESI Baryon Acoustic Oscillations (BAO) data release. In the next two sections, we will provide a brief review of the HDE model and Carrollian geometry. We will then build a Carrollian extension of the HDE model, for which we will run a statistical analysis in Section 5. The cosmological implications of the model will be summarized in Section 6, and we will present our conclusions in Section 7.

2 Brief review of HDE

Consider a spatially flat FLRW universe with line element

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2), \quad (1)$$

where $a(t)$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter. The Friedmann equation can be written as

$$3H^2 = \frac{1}{2\kappa^2} (\rho_m + \rho_{\text{de}}), \quad (2)$$

where $\kappa^2 = 1/16\pi G$, ρ_m is the dust matter density, and ρ_{de} denotes the dark energy (DE) energy density.

In holographic approaches to DE, one starts from the requirement that, in an effective field theory defined in a region of size L , the total vacuum energy should not exceed the mass of a black hole of the same size, implying $\rho_{\text{de}} \lesssim 2\kappa^2 L^{-2}$ [7]. By saturating this constraint, one obtains $\rho_{\text{de}} = 6C^2\kappa^2 L^{-2}$, where C is a dimensionless parameter. In [8], the IR cutoff L is identified with the future event horizon of the FLRW universe, defined as

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}, \quad \dot{R}_h = HR_h - 1, \quad (3)$$

where we have also denoted its derivative which will become useful in the following. With this choice, the DE energy density and DE abundance become

$$\rho_{\text{de}} = 6C^2\kappa^2 R_h^{-2}, \quad \Omega_{\text{de}} \equiv \frac{\rho_{\text{de}}}{\rho_c} = \frac{C^2}{H^2 R_h^2}, \quad (4)$$

where $\rho_c \equiv 6\kappa^2 H^2$ is the critical density. The conservation equation of the DE sector,

$$\dot{\rho}_{\text{de}} + 3H(1 + w_{\text{de}})\rho_{\text{de}} = 0, \quad (5)$$

can be rewritten for the DE equation of state (eos) parameter w_{de} as

$$w_{\text{de}} = -\frac{1}{3C} \left(C + 2\sqrt{\Omega_{\text{de}}} \right), \quad (6)$$

where we have used equation (3). The parameter C controls the DE behavior. For $C = 1$, the model tends asymptotically to the cosmological constant behavior, while for $C \gtrless 1$ the dark energy behaves like quintessence/phantom. Differentiating equation (4) with respect to $\ln a$ one finds the evolution equation for the DE density parameter as

$$\Omega'_{\text{de}} = \Omega_{\text{de}}(1 - \Omega_{\text{de}}) \left(1 + \frac{2}{C} \sqrt{\Omega_{\text{de}}} \right), \quad (7)$$

where a prime denotes derivative with respect to $\ln a$.

3 Carrollian extension of the HDE

Carrollian geometry is a non-Lorentzian geometric structure naturally associated with null hypersurfaces. A Carrollian manifold is characterized by a degenerate metric structure together with a preferred vector field lying in the kernel of that metric. This framework has proved especially useful in the intrinsic description of black-hole horizons and the study of null infinity and null boundaries.

Let $\mathcal{N}$ be a null hypersurface embedded in a Lorentzian spacetime, with induced degenerate metric $h_{\mu\nu}$ and a preferred null generator v^μ satisfying $h_{\mu\nu}v^\nu = 0$. As we have discussed above, the IR cutoff of standard HDE is horizon-based and the future event horizon R_h defines the size of the effective region. Since horizons and null boundaries carry Carrollian structure, it is natural to ask whether the effective cutoff entering the holographic energy density should be sensitive not only to the global horizon scale R_h , but also to the intrinsic geometry of the associated null boundary. In this note we assume that the bulk spacetime is the standard FRW background, while the null boundary geometry associated with the holographic cutoff is allowed to modify the infrared length through a dimensionless deformation factor built from the boundary. At the homogeneous level, the relevant Carrollian scalar is the expansion of the null generators of the future event horizon. For a spherically symmetric null hypersurface with area radius $R_h(t)$ and cuts of area $A = 4\pi R_h^2$, the null expansion is defined intrinsically on the boundary as

$$\Theta_{\text{EH}} \equiv \frac{1}{A} \frac{dA}{d\lambda} = \frac{2}{R_h} \frac{dR_h}{d\lambda}, \quad (8)$$

where λ is a parameter along the null generators. Choosing $\lambda = t$ and using equation (3), one

obtains for the FRW geometry

$$\Theta_{\text{EH}} = 2 \left(H - \frac{1}{R_h} \right). \quad (9)$$

It is convenient to define the dimensionless boundary expansion scalar as

$$X \equiv \frac{\Theta_{\text{EH}}}{2H} = 1 - \frac{1}{HR_h}. \quad (10)$$

The quantity X vanishes when the event horizon is stationary (as in dS spacetime), and measures the normalized rate of change of the horizon area in the general case. Motivated by the above discussions, we promote the future event horizon cutoff to an effective infrared length of the form

$$L_C^{-2} = R_h^{-2} f(X), \quad (11)$$

where L_C is the effective Carrollian infrared cutoff and $f(X)$ is a dimensionless function. The standard HDE model is recovered for $f(X) = 1$, corresponding either to stationary horizon with $X = 0$ or to the limit in which the Carrollian corrections are neglected. For simplicity, in this paper, we will assume a linear model $f(X) = 1 + \alpha X$, where α is a model parameter. With the Carrollian cutoff (11), the DE energy density becomes $\rho_{\text{de}} = 6C^2\kappa^2 R_h^{-2} f(X)$.

4 Cosmological setup

The DE density parameter in the Carrollian extension of the HDE (CHDE) model is given by

$$\Omega_{\text{de}} = \frac{C^2 f(X)}{H^2 R_h^2} = \Omega_{\text{de}}^{(\text{HDE})} f(X). \quad (12)$$

Now, using (3) and (5), one can obtain

$$w_{\text{de}} = -1 + \frac{2}{3}X - \frac{1}{3} \frac{d \ln f}{dX} X' = w_{\text{de}}^{\text{HDE}} - \frac{1}{3} \frac{f'}{f} = -1 + \frac{2}{3}X - \frac{\alpha X'}{3(1 + \alpha X)}. \quad (13)$$

Assuming a flat universe containing dust and DE with separate conservation equation, the evolution of Ω_{de} satisfies the general relation

$$\Omega'_{\text{de}} = \Omega_{\text{de}}(1 - \Omega_{\text{de}}) \left[3 - 2X + \frac{\alpha X'}{1 + \alpha X} \right]. \quad (14)$$

To close the above system, one needs to express X in terms of the DE abundance. It could be easily shown that using equations (12) and (10) one has

$$X = 1 - \frac{\sqrt{\Omega_{\text{de}}}}{C \sqrt{f(X)}}. \quad (15)$$

From the above equation, X could be found in terms of Ω_{de} from the following equation

$$\Omega_{de} = C^2(1 + \alpha X)(1 - X)^2. \quad (16)$$

By defining the redshift as $1 + z = 1/a$, the Friedmann equation can be simplified as

$$h^2 = \frac{\Omega_{m0}(1 + z)^3}{1 - \Omega_{de}(z)}, \quad (17)$$

where we have used the matter conservation, Ω_{m0} is the current value of the matter abundance and $h(z) \equiv H(z)/H_0$ is the dimensionless Hubble parameter. From equation (14), the evolution equation of the DE abundance can also be obtain in terms of redshift as

$$(1 + z) \frac{d\Omega_{de}}{dz} = -\Omega_{de}(1 - \Omega_{de})(3 - 2X) \left[\frac{2 - \alpha + 3\alpha X}{2(1 + \alpha X) - \alpha(1 - X)\Omega_{de}} \right]. \quad (18)$$

5 Statistical setup

In order to constrain the model and cosmological parameters H_0 , Ω_{m0} , C and α , we will use different combinations of the following datasets:

Cosmic Chronometers

The Cosmic Chronometers (CC) is a direct and model-independent method for determining the Hubble parameter $H(z)$ by measuring the differential age evolution of passively evolving, massive early-type galaxies. In this note we employ the 31 CC data points [11] which assumed to be independent.

Pantheon⁺

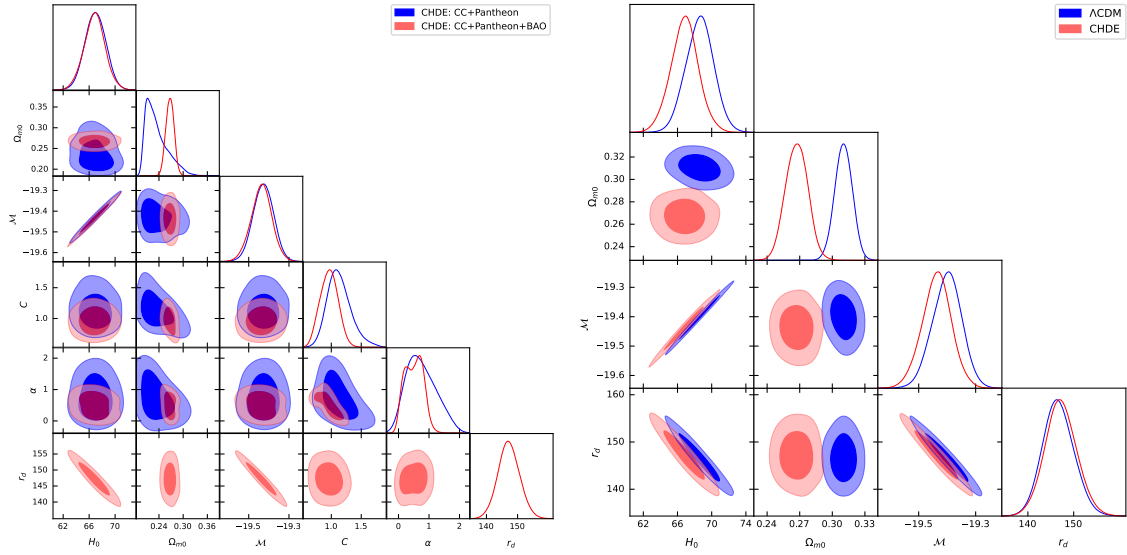
The Pantheon⁺ compilation [12] represents the most updated and homogeneous collection of Type Ia supernova (SN Ia) distance measurements, consists of about 1500 spectroscopically confirmed SNe Ia spanning the redshift range $0.001 < z < 2.26$, combining observations from 18 different surveys. Here we employ the Pantheon⁺ dataset without the SH0ES Cepheid calibration [13], so that the absolute magnitude $\mathcal{M}$ is assumed to be a free parameter and will be inferred from the fitting.

DESI(DR2) BAO

We use the Baryon Acoustic Oscillation (BAO) measurements from the second data release of the Dark Energy Spectroscopic Instrument (DESI DR2) [4]. The dataset includes BAO observables extracted from several tracers of large-scale structure, namely the Bright Galaxy, the Luminous Red Galaxy and the Emission Line Galaxy samples and also quasars, covering the interval $0.2 \lesssim z \lesssim 2.4$. In this note, we will use two different combinations of the above datasets, namely

- CC + Pantheon⁺,
- CC + Pantheon⁺ + BAO.

By maximizing the likelihood function, the best fit values of the parameters H_0 , Ω_{m0} , C , α ,


 Figure 1: The corner plot of the values of parameters with their 1σ and 2σ confidence levels.

r_d and $\mathcal{M}$ can be obtained through the MCMC analysis. In table 1, we have summarized the best-fit values together with their 1σ uncertainties for the Λ CDM and CHDE models. We have also reported the value of the reduced loss function χ_{red}^2 defined as $\chi_{red}^2 = \chi^2/\text{dof}$, where dof is the number of degrees of freedom. It should be noted that the values of this quantity is around unity for all models and combinations, meaning that the fit is statistically consistent with the observational uncertainties.

Model	CHDE		Λ CDM	
	CC+PNT	CC+PNT+BAO	CC+PNT	CC+PNT+BAO
H_0	$66.971^{+1.597}_{-1.600}$	$66.831^{+1.493}_{-1.552}$	$66.549^{+1.618}_{-1.636}$	$68.604^{+1.624}_{-1.654}$
Ω_{m0}	$0.234^{+0.029}_{-0.028}$	$0.267^{+0.010}_{-0.010}$	$0.357^{+0.018}_{-0.018}$	$0.310^{+0.008}_{-0.008}$
C	$1.130^{+0.181}_{-0.189}$	$0.962^{+0.149}_{-0.169}$	—	—
α	$0.743^{+0.520}_{-0.522}$	$0.501^{+0.308}_{-0.343}$	—	—
$\mathcal{M}$	$-19.427^{+0.049}_{-0.049}$	$-19.437^{+0.047}_{-0.049}$	$-19.451^{+0.051}_{-0.051}$	$-19.400^{+0.050}_{-0.051}$
r_d	—	$147.184^{+3.345}_{-3.320}$	—	$146.640^{+3.474}_{-3.359}$
χ_{red}^2	$1.024^{+0.001}_{-0.001}$	$1.025^{+0.002}_{-0.002}$	$1.010^{+0.001}_{-0.001}$	$1.015^{+0.002}_{-0.001}$

 Table 1: Constraints on CHDE and Λ CDM model parameters.

In figure (1) we have plotted corner plots for all the dataset combinations for the CHDE model. Also we have plotted the comparative corner plot of the CHDE model for two dataset combinations. From the corner plots, one can infer that besides strong parameter correlations between H_0 , $\mathcal{M}$ and r_d which is also present in the Λ CDM, model parameters are not strongly correlated.

6 Cosmological implications

Let us now consider cosmological implications of the CHDE model. In the left panel of figure (2) we have plotted the evolution of the Hubble parameter for the Λ CDM and CHDE models. We have also depicted the relative difference between the two models. One can see from the

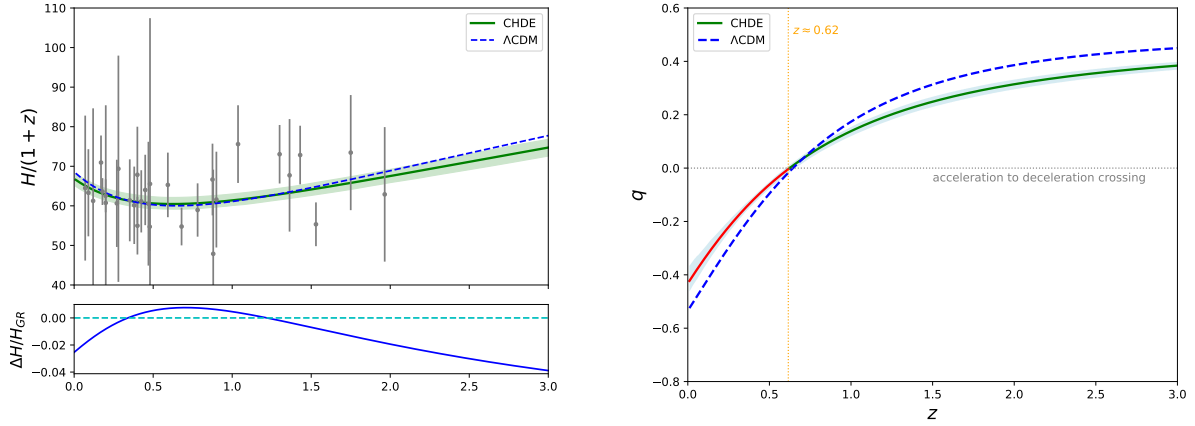


Figure 2: Evolution of the rescaled Hubble parameter $H/(1+z)$ (left) and the deceleration parameter q (right) as functions of redshift for Λ CDM (dashed) and CHDE (solid) models. We have also depicted the relative difference between the two models. We have used the best fit parameters of the full dataset summarized in 1. Shaded area denotes the 1σ error for the CHDE model.

figure that predictions of all the models are very similar. The maximum deviation between the two models is around 0.4%, which occurs at early times. The minimum of the rescaled Hubble diagram denotes the time of deceleration to acceleration phase transition. It can be seen from the figure that the minimum for the Λ CDM model takes place at slightly larger redshifts compared to CHDE model. This indicates that the accelerated expanding era is slightly older in Λ CDM model. In the right panel of figure (2) we have plotted the evolution of the deceleration parameter defined as

$$q = -1 + (1+z) \frac{h'}{h}. \quad (19)$$

One can see from the figure that the late-time behavior of the models are significantly different with each other. Although the Λ CDM model predicts more acceleration at the present time, the CHDE model predicts slightly less acceleration. Also, the deceleration to acceleration phase transition redshift for the CHDE model is $z \approx 0.62$ which is less than the Λ CDM value $z \approx 0.64$. This predicts a slightly older acceleration period for Λ CDM model compared to the CHDE model as we have discussed from the Hubble diagram. In figure (3) we have depicted the evolution of the DE abundance Ω_{de} and the dimensionless boundary expansion scalar X defined in (10) as functions of the redshift z using the best fit values obtained in 1. As shown in the left panel, the DE abundance in the CHDE model is higher than that in the Λ CDM model, indicating a larger DE contribution. However, as we have seen before, due to the non-linear structure of the Friedmann equation in this model, the cosmic acceleration is weaker at late times. Furthermore, the X plot indicates that, at the present epoch, the expansion is stronger than the de Sitter expansion. This behavior corresponds to a larger boundary horizon in the

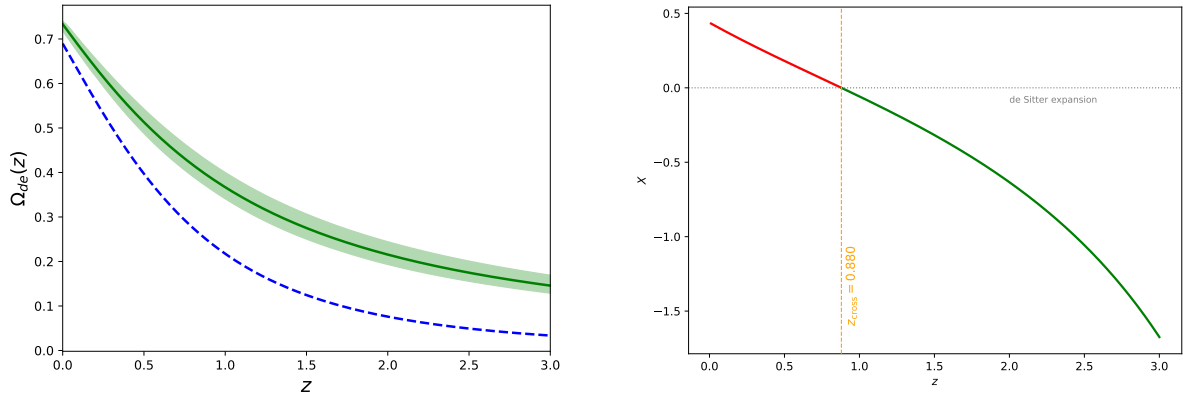


Figure 3: Evolution of the DE abundance Ω_{de} (left) and the dimensionless boundary expansion scalar X (right) as functions of redshift for Λ CDM (dashed) and CHDE (solid) models. We have used the best fit parameters of the full dataset summarized in 1. Shaded area denotes the 1σ error for the CHDE model.

CHDE model than in the Λ CDM model. The transition through the de Sitter limit occurs at $z \simeq 0.88$, indicating that the holographic horizon becomes larger than its de Sitter value at late times.

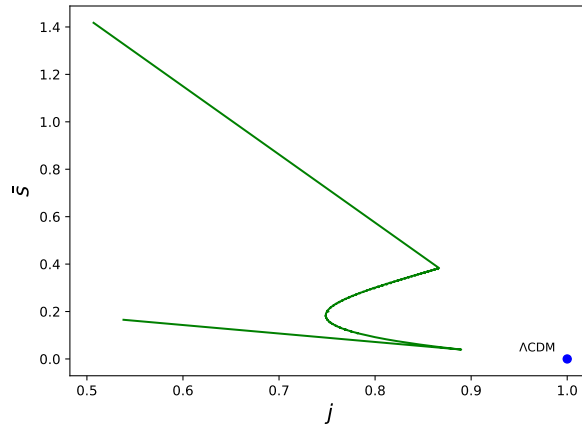


Figure 4: The behavior of statefinder curve in $\bar{s}-j$ plane for CHDE model for the best fit values of the parameters as given by table 1. We have also specified the Λ CDM point.

A well-known diagnostic tool for analyzing the behavior of the DE sector is the statefinder analysis introduced in [14]. In this analysis we use the statefinder pair $\{j, \bar{s}\}$ where j is the jerk parameter and $\bar{s}$ is a combination of the deceleration parameter q and j defined as

$$\bar{s} = \frac{2(j-1)}{3(2q-1)}. \quad (20)$$

For Λ CDM model, the statefinder pair denotes a point $(1, 0)$ in $(j, \bar{s})$ plane. Generally the values $j < 1$ and $\bar{s} > 0$ indicates quintessence-like behavior for the dark energy sector but $j > 1$ and $\bar{s} < 0$ indicate phantom-like behavior. In figure (4), we have depicted the evolution of the statefinder pair for the CHDE model. It is evident that the DE sector has a quintessence-like

behavior. This result is in agreement with the value of C reported in table 1.

7 Conclusions and final remarks

In this paper, we considered a new generalization of the Holographic Dark Energy model by incorporating Carrollian geometric quantities into the boundary structure of the universe. The standard HDE model is based on the introduction of a boundary, usually identified with the future event horizon. As is well known, this boundary is a null hypersurface and is naturally described by Carrollian geometry. It was therefore interesting to ask whether Carrollian quantities can influence the boundary of the universe. We pushed forward this idea and investigated the cosmological implications of this Carrollian extension of the HDE model. We showed that the universe described by CHDE experiences a smaller acceleration at late times and a smaller deceleration at early times compared to the standard Λ CDM model. Furthermore, we found that the behavior of the universe is quintessence-like, which is in agreement with current observational data [15]. In summary, we have shown that the intrinsic geometry of the boundary can have observable cosmological effects at both early and late times.

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Full paper - Poster

Phenomenological Consequences of AdS-Scale Deformations in RPS Holography of Charged AdS Black Holes

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Abstract. We investigate the thermodynamics of restricted Phase Space(RPS) for charged AdS black holes with incorporating the contribution from quantum gravity to the event horizon area for alterations in AdS curvature scale from Holographic viewpoint. We find that AdS scale deformations lead to a large modification in thermodynamic architecture and more clearly increase for large variations of horizon radius. By taking centrality charge as our holographic central parameter and defining an adjoined dimensionless correction namely ϵ for mass, Hawking temperature and chemical potentials, electric potential and heat capacity, Helmholtz free energy and we analyze the effect of that deformation using its plots.

Keywords: charged AdS black holes; Restricted Phase Space Holographic; Thermodynamics; AdS length parameter.

1 Introduction

The thermodynamics of black holes has attracted a lot of attention because it is very important for understanding the nature of quantum gravity. The four laws of black hole thermodynamics, similar to the laws of classical thermodynamics, were established by Hawking, Bekenstein, Bardeen, and Carter [1]. They found that black holes emit radiation and have temperature and entropy. In addition, it was found that entropy is proportional to the surface area of the black hole's event horizon [2, 3].

Visser proposed a new version of extended phase space thermodynamics within the anti-de Sitter/conformal field theory (AdS/CFT) framework [4, 5]. Based on Visser's thermodynamics, which introduced the number of colors and chemical potential as new variables, Gao and Zhao [6] suggested a new formulation for black hole thermodynamics where Newton's constant is treated as a thermodynamic variable. This formulation, known as Restricted Phase Space (RPS) thermodynamics, does away with the pressure-volume (P, V) pair and restores the interpretation of mass as internal energy. In this work, we introduce a deformation parameter that encodes corrections to the AdS length scale in a CFT-inspired thermodynamic framework. In this way, these corrections are reflected in the volume geometry. By introducing a small change in the

AdS radius, the change is considered as follows:

$$\ell \longrightarrow \frac{\ell}{\sqrt{1+\epsilon}}, \quad (1)$$

here, $\epsilon \ll 1$ represents a perturbative parameter. Under this change, the effective AdS length gets modified, and the corresponding black hole thermodynamics receives corrections, and naturally, the central charge is also encoded in the deformed background.

We look at changes in mass, temperature, chemical potentials, electric potential, Helmholtz free energy, and heat capacity as functions of the event horizon. Our goal is to figure out whether quantum geometric effects just shift thermodynamic observables or if they create completely different stability features. Through this comparison, we want to deepen our understanding of holographic black hole thermodynamics and the role of quantum gravity corrections in AdS backgrounds.

In order to study whether quantum-gravity effects leave a phenomenological imprint upon the thermodynamics of the black hole at large scales, we attempt to deform the curvature of AdS in a controlled way, and analyze the deformation of the holographic thermodynamics of charged AdS black holes within the RPS framework. Such provides a new way to link geometric deformations to macroscopic thermodynamic response and, possibly, microscopic quantum gravity effects.

2 RPS Holographic Interpretation

In this section, we establish the thermodynamics of the restricted phase space, which was originally formulated by Gao and Zhao [6]. This formalism is based on the Viscor thermodynamics [4] within the framework of the AdS/CFT correspondence [5]. Notably, this formalism considers a constant cosmological constant and a variable Newton's constant, in which the black hole mass is interpreted as internal energy. The AdS/CFT correspondence states that the partition function of the AdS space is equal to the partition function of the corresponding CFT boundary [6, 7] The central charge of the CFT is defined as follows:

$$C = \frac{\ell^{(d-2)}}{G}. \quad (2)$$

Since the formation of the AdS/CFT symmetry, with the equality $Z_{AdS} = Z_{CFT}$, it can be concluded that the free energy is related to the parameters that characterize black holes [8, 9]:

$$\mu C = M - TS - \Phi Q - \Omega J, \quad (3)$$

where M represents the black hole mass and S corresponds to Bekenstein-Hawking entropy, Φ is the electric potential resulting from the electric charge Q at the black hole event horizon. Ω and J represent the angular velocity and angular momentum, respectively. The Smarr relation in the RPS framework is written as follows [6]:

$$M = TS + \tilde{\Phi} \tilde{Q} + \Omega J + \mu C, \quad (4)$$

here, the electric potential $\tilde{\Phi}$, and the electric charge $\tilde{Q}$, are scaled as follows:

$$\tilde{\Phi} = \frac{\Phi}{\sqrt{C}}, \quad \tilde{Q} = Q\sqrt{C}. \quad (5)$$

The first law of black holes in RPS is stated as follows:

$$dM = TdS + \Omega dJ + \tilde{\Phi} d\tilde{Q} + \mu dC. \quad (6)$$

2.1 Thermodynamic Corrections of Charged AdS Black Holes

In this section, we look at the effects of the AdS length scale changes we introduced on charged AdS black holes in 4-dimensional in the context of finite phase space thermodynamics [6]. The metric for charged AdS black holes is given as follows:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2), \quad (7)$$

here, ℓ is the AdS radius and its metric function is defined as follows:

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} + \frac{r^2}{\ell^2}. \quad (8)$$

These black holes have two horizons, denoted by $(r_{\pm})$. The event horizon, (r_+) , represents the outer radius, while (r_-) indicates the inner horizon. The effect of changing the AdS length scale on the black hole's mass, obtained by solving the equation $f(r_+) = 0$, is as follows:

$$M = \frac{r_+}{2G} \left(1 + \frac{r_+^2(\epsilon + 1)}{\ell^2} + \frac{G\tilde{Q}^2(\epsilon + 1)^3}{Cr_+^2} \right). \quad (9)$$

Bekenstein-Hawking entropy, Hawking temperature, chemical potentials, electric potential and Helmholtz free energy for this charged black hole under this modification are given as follows:

$$S = \frac{\pi r_+^2}{G}, \quad (10)$$

$$T = \frac{dM}{dS}_{\tilde{Q},C} = \frac{-G\ell^2\tilde{Q}^2(\epsilon + 1)^3 + Cr_+^2(\ell^2 + 3r_+^2(\epsilon + 1))}{4\pi C\ell^2(r_+^2)^{3/2}}, \quad (11)$$

$$\mu = \frac{dM}{dC}_{S,\tilde{Q}} = -\frac{\tilde{Q}^2(\epsilon + 1)^3}{2C^2r_+}, \quad (12)$$

$$\tilde{\phi} = \frac{dM}{d\tilde{Q}}_{S,C} = \frac{\tilde{Q}(\epsilon + 1)^3}{Cr_+}, \quad (13)$$

$$F = M - ST = \frac{3G\ell^2\tilde{Q}^2(\epsilon + 1)^3 + Cr_+^2(\ell^2 - r_+^2(\epsilon + 1))}{4CG\ell^2\sqrt{r_+^2}}, \quad (14)$$

Within the framework of standard thermodynamics, if the specific heat capacity at constant pressure ($C_P > 0$), the black hole configuration is thermodynamically stable, whereas ($C_P < 0$) indicates instability. In the current context, this quantity is evaluated by introducing a small change in the AdS curvature radius, which leads to the commonly given form below. This section further examines the phase structure, stability indicators, and the role of the dual field theory (CFT) in shaping the thermodynamic behavior of these black holes, all developed within the framework of holographic dual thermodynamics. The heat capacity is defined as follows:

$$C_P = T(\partial S/\partial T) = \frac{2\pi r_+^2 \left(Cr_+^2(\ell^2 + 3r_+^2(\epsilon + 1)) - G\ell^2\tilde{Q}^2(\epsilon + 1)^3 \right)}{G \left(-C\ell^2r_+^2 + 3Cr_+^4(\epsilon + 1) + 3G\ell^2\tilde{Q}^2(\epsilon + 1)^3 \right)}, \quad (15)$$

2.2 Discussion and result

In this subsection, we analyze the behavior of key thermodynamic quantities within the RPS holographic framework, taking into account the AdS length-scale corrections for a charged AdS black hole, which we calculated earlier. The thermodynamic quantities whose diagrams we plotted include: mass, Hawking temperature, Helmholtz free energy, and heat capacity, all as functions of the horizon radius for three representative values of the correction parameter ϵ (negative, zero, and positive) in terms of the horizon radius. The first two panels show that increasing the horizon radius enhances the separation between the curves corresponding to different ϵ values and highlights the growing effect of thermal fluctuation corrections in the large horizon radius regime. For the first three panels, the corrected case $\epsilon = -0.1$ consistently lies below the reference configuration $\epsilon = 0$, indicating a decrease in mass, temperature, and Helmholtz free energy at a fixed horizon radius. On the other hand, the Helmholtz free energy decreases with increasing horizon radius. The heat capacity for $\epsilon = -0.1$ is higher than in the unmodified case, which indicates a change in the thermodynamic preference of the system and it has little dependence on the modification parameter. At small black hole radii, an instability appears, but the black hole configuration remains thermodynamically stable across the desired horizon radius. The chemical and electric potential diagrams mostly did not show sensitivity to changes in ϵ , so we skipped plotting them.

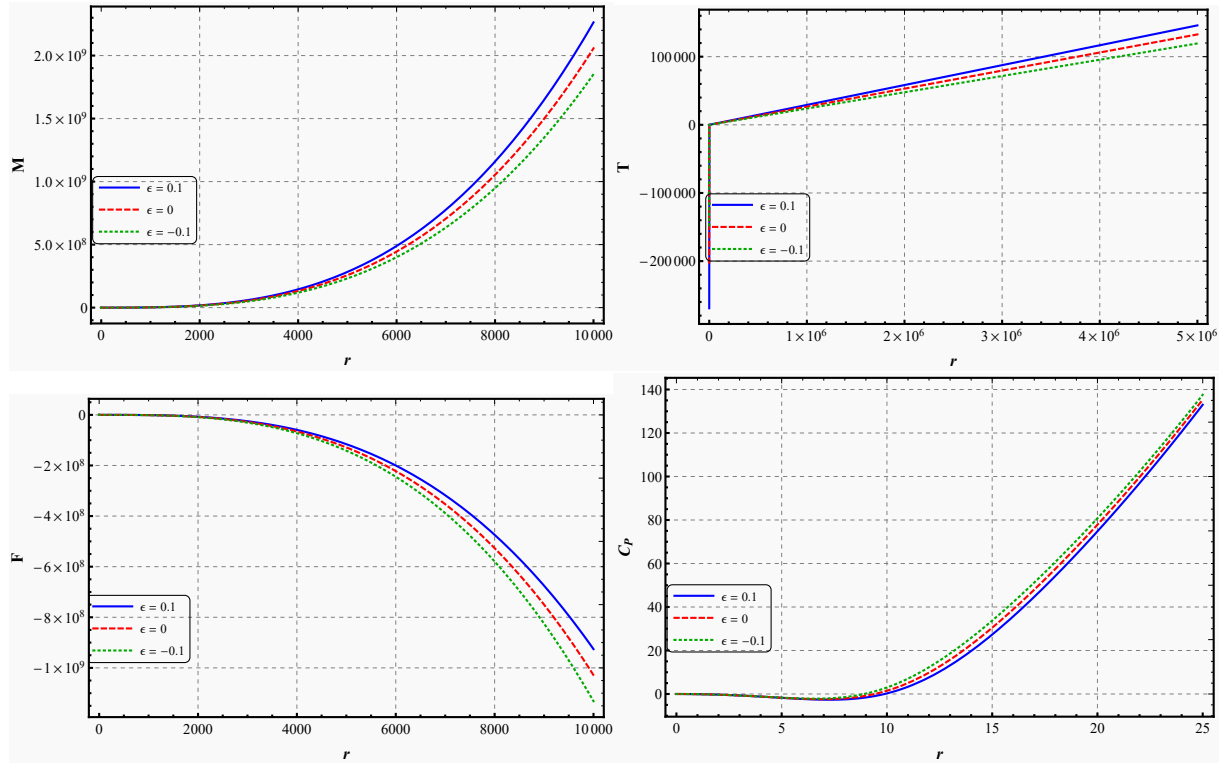


Figure 1: Geometric changes in the AdS length scale on mass, Hawking temperature, Helmholtz free energy, and specific heat for charged AdS black holes.

Figure (1): In this figure, we study the thermodynamic behavior of charged AdS black hole under RPS holographic framework with regard to the horizon radius for some certain parameters of AdS-length deformation. When the horizon radius is larger, the deformation is stronger, which has a greater impact on the mass, Hawking temperature and Helmholtz free energy.

Whereas the heat capacity only responds to faintly qualitatively upon the .

3 Conclusions

In the present work, we examine the thermodynamic consequences of a small AdS-length deformation, $\ell \rightarrow \ell/\sqrt{1+\epsilon}$, on charged AdS black holes in the context of the RPS holographic setting. We find that this deformation becomes more significant for large black holes; mass and Hawking temperature are increased while Helmholtz free energy is decreased. The effect of ϵ does not lead to significant changes in either qualitatively defined stable and unstable regions based on heat capacity; instead, it provides new values for the extent of thermodynamic response. The balance of preferred thermodynamic configurations shifts based on size and electric charge, and the various potentials are somewhat altered. In general, the AdS-length deformation introduces quantitative rather than qualitative effects, though its potential influence on phase transitions, critical behavior and the link to microscopic influences warrant further study. The scope of this and other works in this area provide an interesting look at the possibility for AdS-scale modifications of thermodynamics to manifest at a macroscopic level as well, with issues like the potential microphysical origins of the deformation parameter itself becoming topics for future work.

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The entropic force and real-time dynamics of heavy quarkonia in Yang–Mills–dilaton black holes

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Abstract. We investigate the dissociation of heavy quarkonium in a hyperscaling-violating holographic QCD model by employing both entropic-force and real-time approaches. The effects of the dynamical exponent z , the hyperscaling-violation parameter θ , and the Yang–Mills charge e_3 on the entropy, entropic force, and dissociation time are analyzed. We find that increasing z and e_3 enhances the entropy gradient, leading to a stronger entropic force, whereas increasing θ weakens the entropy gradient despite increasing the entropy itself. Furthermore, the real-time analysis shows that the background parameters significantly modify the thermalization timescale of the falling string.

Keywords: AdS/CFT correspondence, hyperscaling-violation parameter, entropic force, real-time dynamics.

1 Introduction

Heavy quarkonium is one of the most important probes of the strongly interacting quark–gluon plasma (QGP) created in relativistic heavy-ion collisions [1, 2]. Since the seminal proposal of Matsui and Satz [3], the suppression of heavy quarkonium has been regarded as a prominent signature of deconfinement. The dissociation of a heavy quark–antiquark bound state is governed by the screening properties of the hot medium and therefore provides valuable information about the non-perturbative dynamics of QCD.

The gauge/gravity duality offers a powerful framework for investigating strongly coupled gauge theories [4, 5, 6], where conventional perturbative methods and lattice QCD face important limitations, particularly for real-time phenomena. Within this approach, many properties of heavy quarkonium, including the heavy quark potential, screening length, entropy, entropic force, and dissociation dynamics, have been extensively studied using holographic QCD models [7, 9, 10, 11, 8]. Among various holographic backgrounds, geometries with hyperscaling violation provide a more flexible description of strongly coupled plasmas through the dynamical exponent z and the hyperscaling violation parameter θ [12, 13]. These parameters modify the infrared structure of the dual theory and consequently influence the properties of heavy quarkonium in the thermal medium. In this work, we investigate quarkonium dissociation in

a hyperscaling violating holographic QCD model carrying a Yang–Mills charge. We analyze the effects of the dynamical exponent z , the hyperscaling violation parameter θ , and the Yang–Mills charge e_3 on the entropy, entropic force, and real-time dissociation dynamics. Particular attention is paid to the distinct physical information provided by the entropic force and real-time approaches, which probe different aspects of the dissociation process.

2 Setup: Yang–Mills–Dilaton Black Holes

In this section, we briefly review the hyperscaling violating Yang–Mills–dilaton black hole background introduced in Ref. [14]. The corresponding $(n + 1)$ -dimensional action is

$$S = \frac{1}{16\pi G} \int d^{n+1}x \sqrt{-g} \left(R - V(\Phi) - \frac{4}{n-1} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{4} \sum_{i=1}^3 e^{-4\zeta_i \Phi/(n-1)} F_i^2 \right), \quad (1)$$

where Φ is the dilaton field, $V(\Phi)$ is the dilaton potential, and ζ_i denote the dilaton–gauge coupling constants. The corresponding hyperscaling violating Yang–Mills–dilaton black hole solution is

$$ds^2 = r^{2\alpha} \left(-\frac{r^{2z} f(r)}{R^{2z}} dt^2 + \frac{R^2}{r^2 f(r)} dr^2 + r^2 d\Omega_k^2 \right), \quad \alpha := -\theta/(n-1), \quad (2)$$

$$f(r) = 1 + \frac{R^2(n-1)(n-2)}{[(n-1)z - 2\theta][z + n - 3 - \theta]r^2} - \frac{m}{r^{z+n-\theta-1}} + \begin{cases} \frac{(n-1)^2(n-2) R^2 e_3^2 b^{2(z-1)-\frac{2\theta}{n-1}}}{(\theta - n + 1)[(n - z - 3)(n - 1) - (n - 5)\theta] r^{2(z+1)-\frac{4\theta}{n-1}}}, & z \neq n - 3 - \frac{n-5}{n-1}\theta \\ \frac{(n-1)(n-2)R^2 e_3^2 b^{2(z-1)-\frac{2\theta}{n-1}}}{(\theta - n + 1) r^{2(z+1)-\frac{4\theta}{n-1}}} \ln(r), & z = n - 3 - \frac{n-5}{n-1}\theta \end{cases} \quad (3)$$

where $\Phi(r) = \frac{1}{2} \sqrt{(n-\theta-1)[(z-1)(n-1)-\theta]} \ln\left(\frac{r}{b}\right)$. Here, z and θ are the dynamical exponent and hyperscaling violation parameter, respectively, while the integration constant m is determined from the horizon condition $f(r_+) = 0$.

The Hawking temperature associated with the event horizon is

$$T_+ = \frac{(z + n - \theta - 1)r_+^z}{4\pi R^{z+1}} + \frac{(n-1)(n-2)r_+^{z-2}}{4\pi[(n-1)z - 2\theta] R^{z-1}} + \frac{(n-1)(n-2) e_3^2 b^{2(z-1)-\frac{2\theta}{n-1}}}{4\pi(\theta - n + 1) R^{z-1} r_+^{z+2-\frac{4\theta}{n-1}}}, \quad (4)$$

where r_+ is obtained by solving $f(r_+) = 0$. The Yang–Mills charge is

$$Q = \frac{1}{4\pi \sqrt{(n-1)(n-2)}} \int d^{n-1}x \sqrt{Tr \left(F_{\mu\nu}^{(a)} F_{\mu\nu}^{(a)} \right)} = \frac{\omega_{n-1}}{4\pi R^{1-z}} e_3. \quad (5)$$

Since hyperscaling violating geometries provide an effective holographic description valid below a characteristic UV scale, the metric (2) is not assumed to extend to the asymptotic boundary. Instead, the dual field theory is defined at a finite radial cutoff r_F , which plays the role of the UV boundary of the effective theory.

3 Entropic force

In this section, we investigate the effects of the dynamical exponent z , the hyperscaling violation parameter θ , and the Yang–Mills charge e_3 on the entropic force in the hyperscaling violating

Yang–Mills–dilaton background. Following Refs. [10, 15], the entropic force is defined as

$$F = T \frac{\partial \mathcal{S}}{\partial L}, \quad (6)$$

where T is the temperature and L denotes the inter-quark separation. The entropy is obtained from the free energy through

$$\mathcal{S} = -\frac{\partial \mathcal{F}}{\partial T}, \quad (7)$$

where $\mathcal{F}$ is the free energy of the heavy quark-antiquark pair.

To compute these quantities, we consider the Nambu–Goto action

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \mathcal{L} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-g}, \quad (8)$$

where g is the determinant of the induced worldsheet metric. In the string frame, the background metric takes the form

$$ds_s^2 = b^{-\beta} r^{2\alpha+\beta} \left(-\frac{r^{2z} f(r)}{R^{2z}} dt^2 + \frac{R^2}{r^2 f(r)} dr^2 + r^2 d\Omega_k^2 \right), \quad (9)$$

with $\beta := 2\sqrt{(n-\theta-1)[(z-1)(n-1)-\theta]}/3$.

Choosing the static gauge $X^\mu = (\tau, \sigma, 0, 0, r(\sigma))$ the Lagrangian density becomes

$$\mathcal{L} = \sqrt{M(r) + N(r)r'^2}, \quad (10)$$

where

$$M(r) = \frac{r^\gamma}{R^{2z}} f(r), \quad (11)$$

$$N(r) = \frac{r^{\gamma-4}}{R^{2z-2}}, \quad (12)$$

with $\gamma = 4\alpha + 2\beta + 2z + 2$. Two classical string configurations contribute to the Wilson loop [16, 17]. The first is the connected U -shaped string, describing a bound quark–antiquark pair, while the second consists of two disconnected straight strings stretching from the boundary to the horizon and represents two completely screened quarks. The connected solution reaches a turning point r_c , where $r'(r_c) = 0$. Using the conserved Hamiltonian associated with the Nambu–Goto action, one obtains

$$r' = \sqrt{\frac{M^2(r) - M(r)M(r_c)}{N(r)M(r_c)}}, \quad (13)$$

where

$$M_c := M(r_c) = \frac{r_c^\gamma}{R^{2z}} f(r_c). \quad (14)$$

The inter-quark separation is then given by

$$L = 2 \int_{r_c}^{r_F} dr \sqrt{\frac{N(r)M(r_c)}{M^2(r) - M(r)M(r_c)}}, \quad (15)$$

while the free energies of the connected and disconnected configurations take the form

$$\mathcal{F}^{\text{con}} = \frac{1}{\pi\alpha'} \int_{r_c}^{r_F} dr \sqrt{\frac{M(r)N(r)}{M(r) - M(r_c)}}, \quad (16)$$

$$\mathcal{F}^{\text{discon}} = \frac{1}{\pi\alpha'} \int_{r_+}^{r_F} \sqrt{N(r)} dr. \quad (17)$$

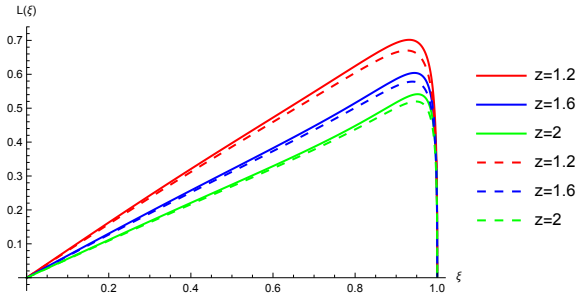


Figure 1: L against ξ in the different z at $\theta = -0.5$ and $e_3 = 1$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

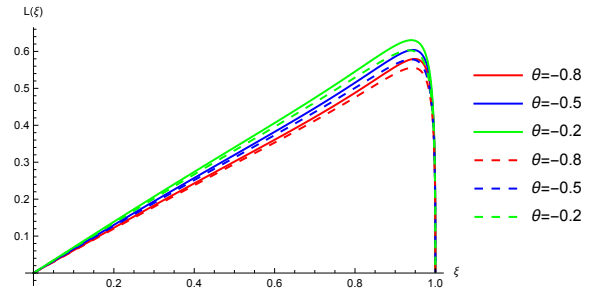


Figure 2: L against ξ in the different θ at $z = 1.6$ and $e_3 = 1$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

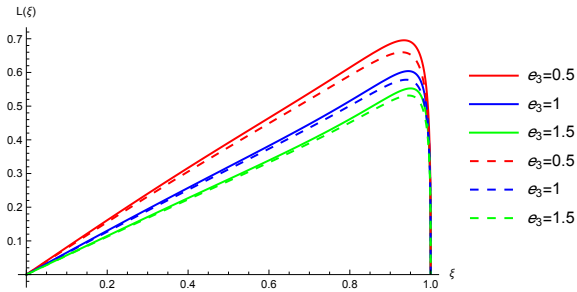


Figure 3: L against ξ in the different e_3 at $\theta = -0.5$ and $z = 1.6$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

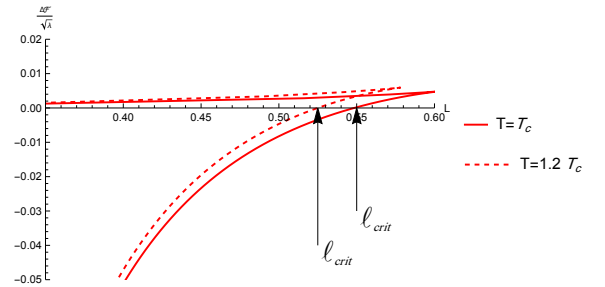


Figure 4: $\Delta\mathcal{F}/\sqrt{\lambda}$ against L at $\theta = -0.5$, $z = 1.6$ and $e_3 = 1$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

Figures 1–3 present the inter-quark separation $L(\xi)$ as a function of $\xi = r_+/r_c$ for different values of the dynamical exponent z , the hyperscaling violation parameter θ , and the Yang–Mills charge e_3 at $T = T_c$ and $T = 1.2T_c$. In all cases, $L(\xi)$ exhibits a characteristic non-monotonic behavior, increasing with ξ , reaching a maximum value L_{max} , and then decreasing as $\xi \rightarrow 1$. The quantity L_{max} represents the largest separation for which a connected U -shaped string solution exists.

Figure 1 shows that increasing the dynamical exponent z decreases L_{max} and shifts its position toward larger values of ξ , indicating that the turning point of the string moves closer to the horizon. A similar behavior is observed for increasing the Yang–Mills charge e_3 , as illustrated in Fig. 3. In both cases, the connected string becomes less stable and the maximum quarkonium size is reduced.

In contrast, Fig. 2 shows that increasing the hyperscaling violation parameter θ increases $L_{\max}$, allowing the connected string to survive up to larger inter-quark separations. For all parameter choices, increasing the temperature suppresses $L(\xi)$ and reduces $L_{\max}$, reflecting the stronger color screening in the hot plasma.

Before discussing the entropy, it is useful to distinguish between the maximum separation $L_{\max}$ and the critical separation ℓ_{crit} . The latter is determined by the free-energy difference

$$\Delta\mathcal{F} = \mathcal{F}^{\text{con}} - \mathcal{F}^{\text{discon}}, \quad (18)$$

where $\mathcal{F}^{\text{con}}$ and $\mathcal{F}^{\text{discon}}$ are given by Eqs. (16) and (17), respectively. The critical separation is defined by the condition $\Delta\mathcal{F} = 0$, which marks the transition between the connected and disconnected string configurations [11].

For $L < \ell_{\text{crit}}$, the connected configuration has the lower free energy and describes a stable quarkonium bound state, whereas for $L > \ell_{\text{crit}}$ the disconnected configuration becomes thermodynamically favored, corresponding to complete color screening in the plasma. Unlike $L_{\max}$, which is determined solely by the existence of connected string solutions, ℓ_{crit} is obtained from a thermodynamic criterion. As shown in Fig. 4, increasing the temperature shifts ℓ_{crit} toward smaller values, indicating that thermal effects promote quarkonium dissociation. Using Eq. (7), the entropy of the heavy quark–antiquark pair is obtained from the free energy. For the connected and disconnected string configurations, the entropy is given by

$$\mathcal{S}^{\text{con}} = -\frac{\sqrt{\lambda}}{2\pi} \int_{r_c}^{r_F} dr \frac{(M'N + MN')(M - M_c) - MN(M' - M'_c)}{\sqrt{MN(M - M_c)^3}}, \quad (19)$$

$$\mathcal{S}^{\text{discon}} = \frac{r_+^{(\gamma-4)/2}}{\pi\alpha'R^{z-1}} \Theta(L - \ell_{\text{crit}}), \quad (20)$$

where the derivatives are taken with respect to the horizon radius r_+ .

The entropy exhibits two distinct regimes separated by the critical distance ℓ_{crit} . For $L < \ell_{\text{crit}}$, the entropy is determined by the connected string configuration and increases monotonically with the inter-quark separation. For $L > \ell_{\text{crit}}$, the disconnected configuration dominates and the entropy approaches a constant value independent of L . Consequently, the entropy develops a discontinuity at ℓ_{crit} , reflecting the first-order transition between the connected and disconnected string configurations.

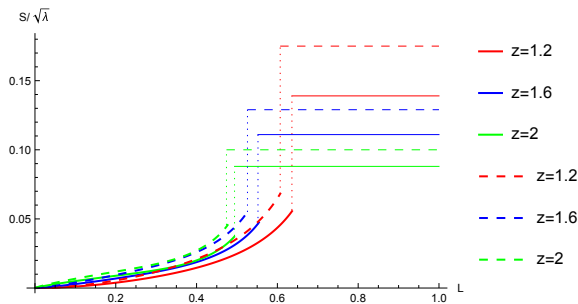


Figure 5: $S/\sqrt{\lambda}$ against L in the different z at $\theta = -0.5$ and $e_3 = 1$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

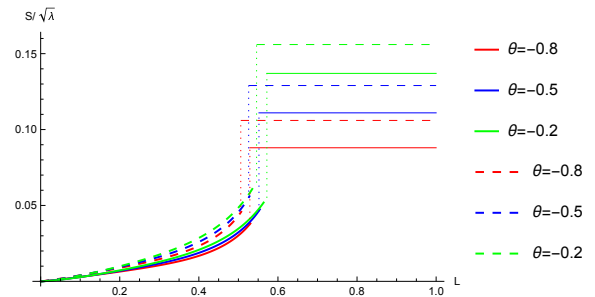


Figure 6: $S/\sqrt{\lambda}$ against L in the different θ at $z = 1.6$ and $e_3 = 1$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

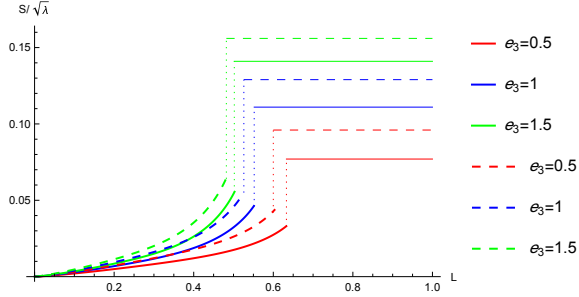


Figure 7: $S/\sqrt{\lambda}$ against L in the different e_3 at $\theta = -0.5$ and $z = 1.6$. The solid and dashed lines represent $T = T_c$ and $T = 1.2T_c$, respectively.

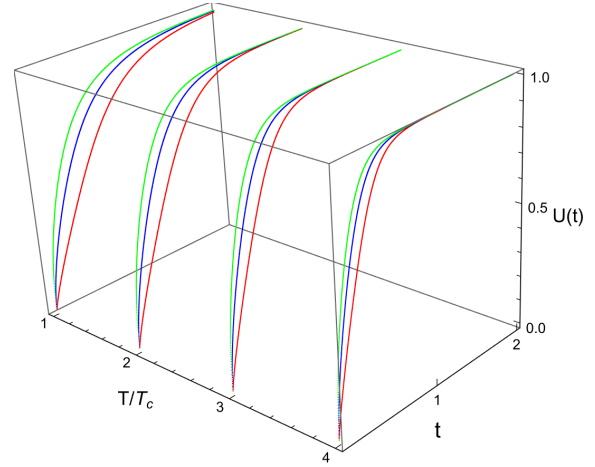


Figure 8: The string motion from the boundary to the horizon.

Figures 5–7 present the entropy of the heavy quark–antiquark pair for different values of the parameters z , θ , and e_3 . In all cases, the entropy increases with the inter-quark separation and eventually saturates at large distances. Since the entropic force is proportional to the entropy gradient, Eq. (6), the slope of the entropy curves determines the thermodynamic tendency toward quarkonium dissociation. Figure 5 shows that increasing the dynamical exponent z enhances both the entropy and its slope in the connected phase. Consequently, the entropic force becomes stronger, indicating an enhanced thermodynamic tendency toward quarkonium dissociation. Higher temperatures further increase the entropy and its saturation value. Figure 6 illustrates the effect of the hyperscaling violation parameter θ . Although increasing θ raises the entropy over the entire range of L , it reduces the slope of the entropy curves in the connected phase. Therefore, the entropic force decreases with increasing θ , implying a weaker thermodynamic tendency toward quarkonium dissociation. As expected, increasing the temperature shifts the entropy curves upward and increases their saturation value. Figure 7 shows that the Yang–Mills charge e_3 has a similar effect to the dynamical exponent. Larger values of e_3 increase both the entropy and its slope, leading to a stronger entropic force and consequently a stronger thermodynamic tendency toward quarkonium dissociation. As in the previous cases, increasing the temperature enhances the entropy and raises its saturation value.

4 Real-Time Dynamics

Unlike the entropic-force approach, which provides an equilibrium criterion for quarkonium dissociation, the real-time approach describes the dynamical evolution of the system. In the holographic picture, a heavy quark–antiquark pair created on the boundary is connected by a fundamental string extending into the bulk geometry. As the string falls toward the black-hole horizon under the gravitational attraction of the background, the dual quarkonium state progressively thermalizes with the plasma. At late times the configuration approaches two straight strings ending on the horizon, corresponding to two deconfined heavy quarks. Therefore, the dissociation (thermalization) time can be identified with the time required for the falling string to approach the horizon [18, 8, 19]. The string dynamics is governed by the Nambu–Goto action

$$S = -\frac{1}{2\pi\alpha'} \int dt dx b^{-\beta} r^{2\alpha+\beta} \sqrt{\frac{r^{2z+2}}{R^{2z}} f(r) + \frac{r^{2z-2}}{R^{2z-2}} \dot{r}^2 - \frac{R^2}{f(r)} \dot{r}^2}, \quad (21)$$

which leads to the corresponding equation of motion. Solving the full two-dimensional equation is numerically demanding, particularly for small interquark separations where the spatial derivative becomes significant, and the near-horizon coordinate singularity further reduces numerical stability. On the other hand, if we concentrate on the large inter-quark distances L , we can approximate the string profile by a straight line hanging towards the black hole horizon. Since quarkonium dissociation occurs for sufficiently large interquark distances, the string can be approximated by a straight configuration, $r = r(t)$, allowing the spatial derivative to be neglected. Under this approximation the action and the equation of motion simplify considerably.

$$\partial_t \left(\frac{R^2 r^{2\alpha+\beta} \dot{r}}{f(r) \sqrt{\frac{r^{2z+2}}{R^{2z}} f(r) - \frac{R^2}{f(r)} \dot{r}^2}} \right) + \partial_r \left(r^{2\alpha+\beta} \sqrt{\frac{r^{2z+2}}{R^{2z}} f(r) - \frac{R^2}{f(r)} \dot{r}^2} \right) = 0. \quad (22)$$

Time-translation symmetry implies a conserved energy, from which both the string velocity and the initial energy are determined by the boundary condition $\dot{r}(t=0) = 0$. The simplified equation is solved numerically after introducing the dimensionless coordinate $U = r_+/r$, where $U = 0$ corresponds to the boundary and $U = 1$ to the horizon. The resulting trajectories are shown in Figs. 8–11.

Figure 8 provides a three-dimensional visualization of the string evolution as a function of the normalized temperature T/T_c . As the temperature increases, the string approaches the near-horizon region more rapidly, indicating a shorter thermalization (dissociation) time.

In Figs. 9–11, in all cases the string accelerates rapidly after leaving the boundary and then approaches the horizon asymptotically. Increasing the dynamical exponent z , the hyperscaling-violation parameter θ , or the Yang–Mills charge e_3 causes the string to reach the near-horizon region more rapidly, indicating an earlier thermalization of the dual quarkonium state. Among these parameters, the effect of z and e_3 is considerably stronger, whereas the influence of θ remains comparatively weak. Near the horizon, the late-time motion follows

$$r - r_+ \sim e^{-4\pi T t}, \quad (23)$$

showing that the horizon is reached only asymptotically. Consequently, a finite dissociation time is defined by introducing an infrared cutoff ϵ_{IR} outside the horizon. The dissociation time is then obtained from

$$t_D = \int_{\epsilon_{\text{UV}}}^{1-\epsilon_{\text{IR}}} \frac{R^{z+1} U^{z-1} dU}{r_+^z f(U) \sqrt{1 - \frac{r_+^\gamma U^{-\gamma} f(U)}{4\pi^2 \alpha'^2 R^{2z} E^2}}}. \quad (24)$$

The numerical results are presented in Figs. 12–14. The dissociation time decreases monotonically with temperature, reflecting the faster thermalization of quarkonium in hotter plasmas. Increasing the dynamical exponent z systematically shortens the dissociation time, indicating

that stronger Lifshitz anisotropy enhances medium-induced dissociation. A similar but weaker behavior is observed for the hyperscaling-violation parameter θ , while the Yang–Mills charge e_3 produces a pronounced reduction of t_D . Overall, all three parameters accelerate the infall of the string toward the horizon, leading to faster thermalization and a shorter lifetime of heavy quarkonium in the strongly coupled plasma.

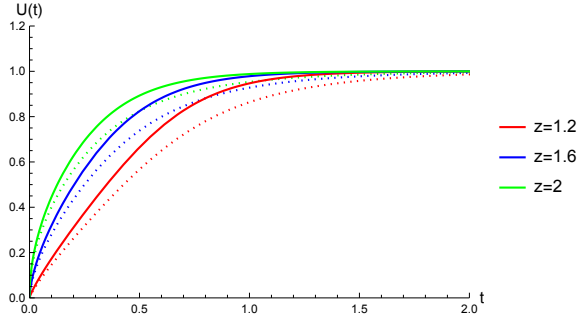


Figure 9: The string motion from the boundary to the horizon in the different z at $\theta = -0.5$ and $e_3 = 1$.

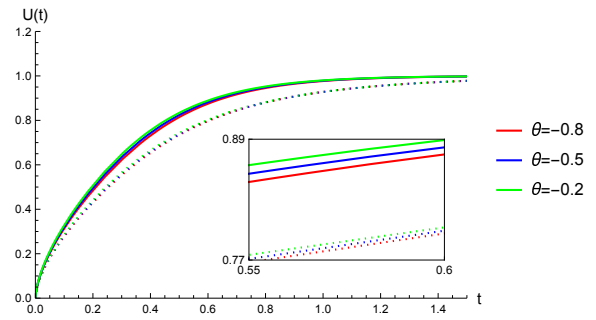


Figure 10: The string motion from the boundary to the horizon in the different θ at $z = 1.6$ and $e_3 = 1$.

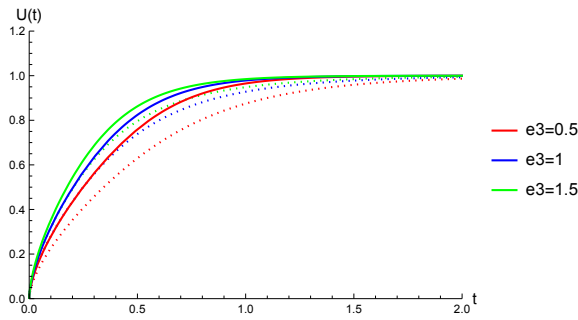


Figure 11: The string motion from the boundary to the horizon in the different e_3 at $\theta = -0.5$ and $z = 1.6$.

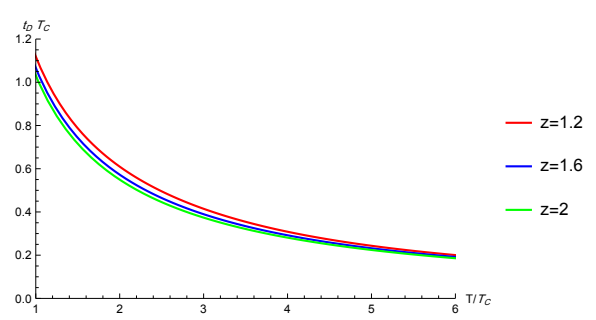


Figure 12: The quarkonium dissociation time as a function of temperature in the different z at $\theta = -0.5$ and $e_3 = 1$.

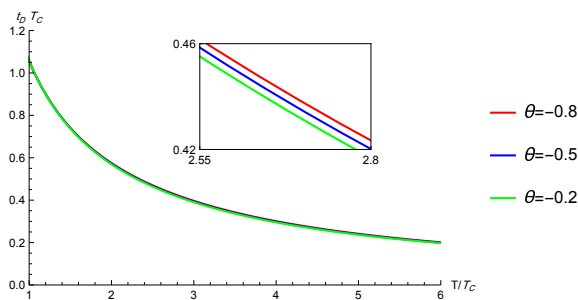


Figure 13: The quarkonium dissociation time as a function of temperature in the different θ at $z = 1.6$ and $e_3 = 1$.

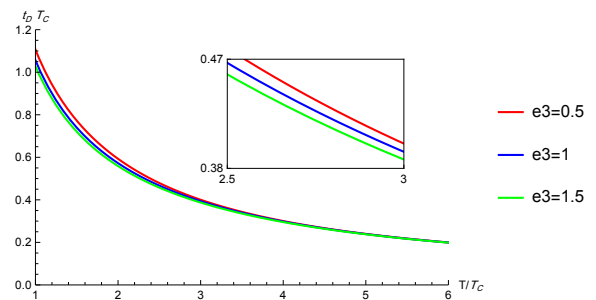


Figure 14: The quarkonium dissociation time as a function of temperature in the different e_3 at $\theta = -0.5$ and $z = 1.6$.

5 Conclusion

In this work, we investigated the influence of the dynamical exponent z , the hyperscaling violation parameter θ , and the Yang–Mills charge e_3 on quarkonium dissociation in hyperscaling-

violating Yang–Mills–dilaton black hole backgrounds by employing two complementary holographic approaches.

Within the entropic-force framework, our numerical analysis shows that increasing z and e_3 enhances the entropy gradient and therefore strengthens the entropic force, leading to a stronger thermodynamic tendency toward quarkonium dissociation. In contrast, although increasing θ raises the entropy itself, it reduces the slope of the entropy curves and consequently weakens the entropic force. These results indicate that the entropy magnitude alone is not sufficient to characterize the dissociation tendency; rather, the entropy gradient plays the decisive role.

We also investigated the real-time dynamics of a falling string and evaluated the corresponding equilibration (dissociation) time. We found that increasing z , θ , or e_3 systematically reduces the equilibration time, implying a faster approach of the string toward the black-hole horizon. This result characterizes the dynamical relaxation of the system after the string evolution has started.

The two approaches probe different aspects of quarkonium physics. The entropic-force analysis describes the thermodynamic stability of the bound state through equilibrium quantities, whereas the real-time calculation determines the dynamical equilibration timescale of the evolving string. Therefore, these approaches are complementary.

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Full paper - Presentation

Thermodynamics and Phase Structure of Charged Simpson–Visser–AdS Black Holes

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Abstract. We investigate the thermodynamic behaviour of charged Simpson–Visser black holes in anti-de Sitter (AdS) spacetime within the framework of extended phase space thermodynamics, where the cosmological constant is interpreted as thermodynamic pressure. The Simpson–Visser regularization parameter a removes the central singularity and modifies the thermodynamic properties of the black hole. We derive the entropy from the first law of thermodynamics, obtaining a result that differs from the standard Bekenstein–Hawking area law while remaining fully thermodynamically consistent. The corresponding equation of state exhibits Van der Waals–type critical behaviour, with the regularization parameter shifting the critical temperature, pressure, and thermodynamic volume. In particular, the universal critical ratio is no longer constant but explicitly depends on a , indicating a deformation of the standard Reissner–Nordström–AdS thermodynamic universality. The Gibbs free energy develops the characteristic swallowtail structure, confirming a first-order phase transition between small and large black holes, while the heat-capacity analysis identifies the stable and unstable thermodynamic branches and locates the second-order transition points. Our results show that the Simpson–Visser parameter acts as a geometric deformation that quantitatively shifts the critical quantities and stability regions without changing the overall thermodynamic phase structure.

Keywords: Black Hole Thermodynamics; Extended Phase Space; Simpson Visser Regular Black Holes; Holographic Principle; Phase Transitions; Van der Waals Criticality.

1 Introduction

Black holes, regarded as thermodynamic systems, provide a unique window into quantum gravity [1, 2]. The AdS/CFT correspondence further strengthened this connection by relating gravitational dynamics in anti-de Sitter (AdS) spacetime to strongly coupled gauge field theories [3]. In particular, it established that thermodynamic phase transitions of AdS black holes correspond to thermal phase transitions in the dual conformal field theory (CFT) defined on the AdS boundary [4]. Consequently, the thermodynamic investigation of AdS black holes has become not only a fundamental problem in gravitational physics but also a powerful tool for exploring strongly coupled quantum field theories.

Classical general relativity, however, predicts the existence of curvature singularities at the cen-

ters of black holes, where the classical description of spacetime breaks down. Regular black hole models provide an elegant resolution to this problem by replacing the singularity with a regular finite core.

Among the various regular black hole models, the Simpson–Visser (SV) geometry offers one of the simplest and most elegant constructions. By replacing the radial coordinate r with $\sqrt{r^2 + a^2}$, the central curvature singularity is removed while the solution continuously reduces to the standard Reissner–Nordström (RN) geometry in the limit $a \rightarrow 0$ [5].

Recent investigations have explored the thermodynamic properties of Simpson–Visser black holes in AdS spacetime [6]. Nevertheless, a comprehensive holographic thermodynamic analysis of the charged Simpson–Visser–AdS black hole within the framework of extended phase space thermodynamics is still lacking. In this formalism, the cosmological constant is interpreted as thermodynamic pressure P , while the black-hole mass is identified with the thermodynamic enthalpy [7, 8]. This formulation leads to an equation of state analogous to that of a Van der Waals fluid and gives rise to a rich thermodynamic phase structure.

Recent studies have also demonstrated that thermodynamic geometry provides an efficient framework for understanding the microscopic interactions and phase structure of modified black holes [9, 10].

In the present work, we investigate the holographic thermodynamics and phase structure of charged Simpson–Visser–AdS black holes within the extended phase space formalism. We derive the complete set of thermodynamic quantities, including the temperature, entropy (obtained from the first law of thermodynamics), thermodynamic volume, and electric potential. Our primary objective is to determine how the Simpson–Visser regularization parameter a modifies the thermodynamic behaviour of the system. We demonstrate that, although the overall critical behaviour remains qualitatively similar to that of the RN–AdS black hole, the universal ratio $P_c V_c / T_c$ is modified, thereby breaking its exact universality. We further analyse the Gibbs free energy and the heat capacity to characterize the first- and second-order phase transitions as well as the thermodynamic stability regions. Our results show that the parameter a acts as a deformation parameter that shifts the critical points and stability boundaries without changing the overall thermodynamic phase structure.

2 Geometrical and Thermodynamic Preliminaries

In this section, we briefly review the geometry of the charged Simpson–Visser–AdS black hole and summarize the thermodynamic quantities required for the subsequent holographic analysis. Throughout this work we employ natural units $G = \hbar = c = k_B = 1$, unless otherwise stated.

2.1 Charged Simpson–Visser–AdS Geometry

The charged Simpson–Visser black hole provides a regular deformation of the Reissner–Nordström spacetime in which the central curvature singularity is removed through the regularization parameter a , replacing the standard areal radius r by $\sqrt{r^2 + a^2}$ [5]. In the presence of a negative cosmological constant, the corresponding AdS solution is described by the static and spherically symmetric metric [5, 6]

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + (r^2 + a^2) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

with the lapse function

$$f(r) = 1 - \frac{2Mr^2}{(r^2 + a^2)^{3/2}} + \frac{Q^2 r^2}{(r^2 + a^2)^2} + \frac{r^2 + a^2}{\ell^2}, \quad (2)$$

where M is the ADM mass, Q is the electric charge, a denotes the Simpson–Visser regularization parameter, and ℓ is the AdS curvature radius related to the cosmological constant through $\Lambda = -3/\ell^2$.

The parameter a has the dimension of length and characterizes the size of the regular core that replaces the classical curvature singularity at the black-hole center. Therefore, it controls the deviation from the standard Reissner–Nordström–AdS geometry in the strong-gravity regime while preserving the asymptotic AdS structure. Larger values of a correspond to stronger regularization effects near the center of the spacetime. In the limiting case $a \rightarrow 0$, the regular core disappears and the geometry smoothly reduces to the standard Reissner–Nordström–AdS (RN–AdS) black hole [5]. The event horizon is determined by the largest positive root of $f(r_+) = 0$, which yields the mass parameter [6]

$$M = \frac{(r_+^2 + a^2)^{3/2}}{2r_+^2} \left[1 + \frac{Q^2 r_+^2}{(r_+^2 + a^2)^2} + \frac{r_+^2 + a^2}{\ell^2} \right]. \quad (3)$$

As will be discussed in the next subsection, this mass plays the role of thermodynamic enthalpy once the cosmological constant is identified with pressure in the extended phase space formalism [7, 8].

The horizon area is given by

$$A = 4\pi (r_+^2 + a^2), \quad (4)$$

which explicitly depends on a and causes several thermodynamic quantities to deviate from their RN-AdS counterparts when $a \neq 0$ [6]. The surface gravity at the Killing horizon is

$$\kappa = \frac{1}{2} f'(r) \Big|_{r=r_+}, \quad (5)$$

from which the Hawking temperature follows as [6]

$$T = \frac{\kappa}{2\pi} = \frac{r_+}{4\pi} \left[\frac{1}{r_+^2 + a^2} - \frac{Q^2}{(r_+^2 + a^2)^2} + \frac{3}{\ell^2} \right]. \quad (6)$$

This temperature expression shows that the regularization parameter modifies both the gravitational and electromagnetic contributions. For sufficiently large a , the temperature profile shifts noticeably relative to the RN-AdS case, leading to significant changes in the critical behaviour and phase structure analysed in the following sections.

2.2 Extended Thermodynamic Formalism

Within the extended phase space formalism the cosmological constant is promoted to a thermodynamic variable and interpreted as the pressure of the background spacetime rather than as a fixed parameter of the theory [7, 8]. This approach enriches the thermodynamic phase space and allows black hole phase transitions to be studied in direct analogy with ordinary thermodynamic systems.

The thermodynamic pressure is identified as

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi\ell^2}, \quad (7)$$

where $\Lambda = -3/\ell^2$. Consequently, the mass parameter in Eq. (3) is no longer interpreted as the internal energy but rather as the thermodynamic enthalpy [7, 8],

$$M \equiv H(S, P, Q). \quad (8)$$

This identification guarantees the consistency of the extended first law and provides the natural thermodynamic description for AdS black holes.

The Hawking temperature follows directly from the surface gravity,

$$T = \frac{f'(r_+)}{4\pi}, \quad (9)$$

which, upon substituting the metric function (2), yields [6]

$$T = \frac{r_+}{4\pi} \left[\frac{1}{r_+^2 + a^2} - \frac{Q^2}{(r_+^2 + a^2)^2} + \frac{3}{\ell^2} \right]. \quad (10)$$

Unlike the RN-AdS solution, the SV regularization parameter modifies both the geometric and electromagnetic contributions, shifting the temperature profile and significantly altering the thermodynamic phase structure. The entropy cannot be obtained directly from the standard Bekenstein–Hawking area law [1, 2] because the Simpson–Visser regularization modifies the near-horizon geometry and consequently changes the entropy-area relation [6].

Instead, it is determined by integrating the first law while keeping the parameter a fixed, which ensures thermodynamic consistency for the regular SV geometry [6]. Since the presence of the Simpson–Visser regularization modifies the horizon geometry, the entropy cannot be directly identified with one quarter of the horizon area. Instead, it is obtained by integrating the first law of black-hole thermodynamics while keeping the electric charge and thermodynamic pressure fixed,

$$dS = \frac{1}{T} \left(\frac{\partial M}{\partial r_+} \right) dr_+, \quad (11)$$

Substituting Eqs. (3) and (9) into the above relation and performing the integration analytically leads to the following expression for the entropy.

$$S = \pi \left[r_+ \sqrt{r_+^2 + a^2} + a^2 \ln \left(r_+ + \sqrt{r_+^2 + a^2} \right) \right], \quad (12)$$

which reduces to the standard $S = \pi r_+^2$ in the limit $a \rightarrow 0$.

The thermodynamic volume, conjugate to the pressure, is defined as [7]

$$V = \left(\frac{\partial M}{\partial P} \right)_{S,Q} = \frac{4\pi}{3} (r_+^2 + a^2)^{3/2}, \quad (13)$$

and the electric potential measured at infinity relative to the horizon is

$$\Phi = \frac{Q}{\sqrt{r_+^2 + a^2}}. \quad (14)$$

The complete thermodynamic description is therefore governed by the variables

$$(M, T, S, P, V, Q, \Phi),$$

which satisfy the extended first law [8]

$$dM = T dS + V dP + \Phi dQ, \quad (15)$$

and the corresponding Smarr relation [8]

$$M = 2TS - 2PV + \Phi Q. \quad (16)$$

Equations (15) and (16) ensure that all thermodynamic quantities are mutually consistent within the extended phase space formalism.

To investigate the thermodynamic phase structure, we introduce the Gibbs free energy,

$$G = M - TS, \quad (17)$$

which is the appropriate thermodynamic potential in the canonical ensemble with fixed pressure and electric charge. The behaviour of G as a function of temperature determines the global thermodynamic stability and signals first order phase transitions through the characteristic swallowtail structure, a well known feature in AdS black hole thermodynamics [8]. The Gibbs free energy therefore plays a central role in the analysis of the complete phase structure of the charged Simpson–Visser–AdS black hole, presented in the following section.

3 Equation of State, Criticality and Phase Structure

In this section, we present the core thermodynamic results for the charged Simpson-Visser-AdS black hole. Using the extended phase space formalism developed in the previous section, we derive the equation of state and analyse its critical behaviour, revealing a Van der Waals like phase structure. We then investigate the Gibbs free energy to identify first order phase transitions through the swallowtail characteristic, and finally examine the heat capacity to determine the local thermodynamic stability and locate second order phase transitions. Throughout this analysis, we focus on how the SV regularization parameter a modifies the critical quantities, the coexistence regions, and the stability conditions compared to the standard RNAdS case.

3.1 Equation of State and Critical Behaviour

The thermodynamic behaviour of the charged Simpson-Visser-AdS black hole can be investigated within the framework of extended black hole thermodynamics, where the cosmological constant is interpreted as a thermodynamic pressure according to Eq. (7). Combining the Hawking temperature given by Eq. (10) with the pressure definition allows one to express the pressure as an explicit function of the horizon radius and temperature. The resulting equation of state takes the form [7, 8]

$$P = \frac{T}{2\sqrt{\left(\frac{3V}{4\pi}\right)^{2/3} - a^2}} - \frac{1}{8\pi\left(\frac{3V}{4\pi}\right)^{2/3}} + \frac{Q^2}{8\pi\left(\frac{3V}{4\pi}\right)^{4/3}}. \quad (18)$$

Equation (18) explicitly expresses the thermodynamic pressure as a function of the thermodynamic volume, making it directly suitable for constructing the $P - V$ isotherms presented in Fig. 1. which reduces to the familiar Reissner-Nordström-AdS equation of state when the regularization parameter vanishes ($a \rightarrow 0$). The presence of the parameter a modifies both the attractive and repulsive contributions of the effective thermodynamic interaction, leading to quantitative changes in the phase structure while preserving the general Van der Waals behaviour.

Having established the thermodynamic equation of state, we now investigate the critical behaviour of the charged Simpson-Visser-AdS black hole. Similar to ordinary thermodynamic systems, the critical point corresponds to the inflection point of the isothermal curves in the $P - V$ plane and separates the single-phase region from the coexistence region of small and large black holes. The critical point is determined by the inflection point of the isothermal curves in the $P - V$ diagram. Since the thermodynamic volume is a monotonic function of the horizon radius, the criticality conditions can equivalently be written as $\left(\frac{\partial P}{\partial r_+}\right)_T = 0$ and $\left(\frac{\partial^2 P}{\partial r_+^2}\right)_T = 0$, which are completely equivalent to the conventional conditions $\left(\frac{\partial P}{\partial V}\right)_T = 0$ and $\left(\frac{\partial^2 P}{\partial V^2}\right)_T = 0$.

So we have $r_c = \sqrt{6}Q$, while the corresponding critical temperature and pressure become

$$T_c = \frac{1}{3\sqrt{6}\pi Q} - \frac{Q}{2\pi(a^2 + 6Q^2)}, \quad (19)$$

$$P_c = \frac{1}{96\pi Q^2} - \frac{Q^2}{8\pi(a^2 + 6Q^2)^2}. \quad (20)$$

Unlike the RN-AdS black hole, the universal ratio is no longer constant and explicitly depends on the regularization parameter,

$$\frac{P_c V_c}{T_c} = \frac{3}{8} \left(\frac{6Q^2}{6Q^2 + a^2} \right)^{3/2}, \quad (21)$$

indicating that the Simpson–Visser parameter modifies the microscopic thermodynamic structure responsible for the critical behaviour.

In the limiting case $a = 0$, all thermodynamic quantities continuously recover the well known RN-AdS results,

$$r_c = \sqrt{6} Q, \quad T_c = \frac{1}{3\sqrt{6}\pi Q}, \quad P_c = \frac{1}{96\pi Q^2}, \quad \frac{P_c V_c}{T_c} = \frac{3}{8}, \quad (22)$$

confirming the consistency of the present formulation.

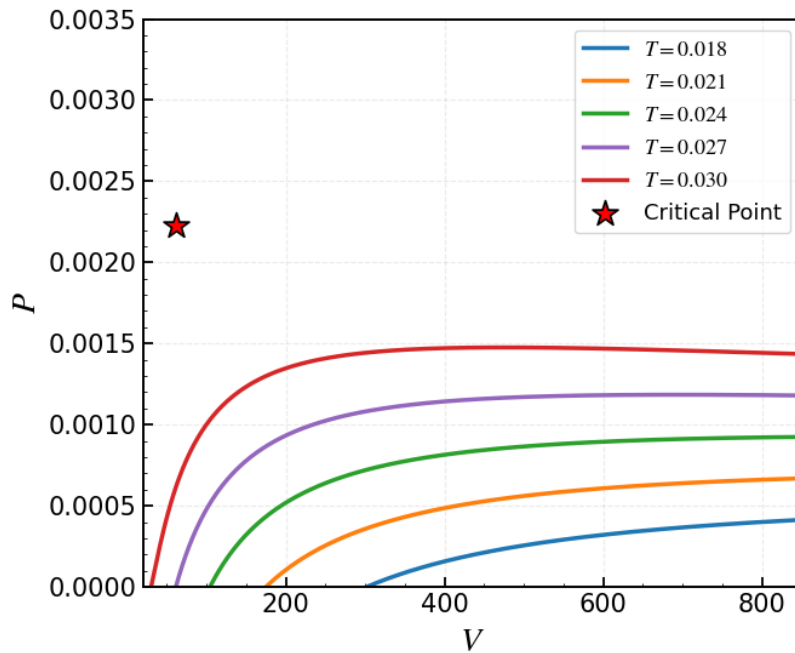


Figure 1: Equation of state of the charged Simpson–Visser–AdS black hole in the extended phase space. The black curves correspond to different isotherms, while the red curve represents the critical isotherm. Below the critical temperature the oscillatory behaviour is analogous to the Van der Waals fluid and indicates the coexistence of small and large black hole phases.

Figure 1 shows the P – V isotherms of the charged Simpson–Visser–AdS black hole. Above the critical temperature, only a single stable phase exists. At the critical temperature, the isotherm exhibits an inflection point, while below the critical temperature the oscillatory behaviour indicates a first-order phase transition between small and large black holes.

The influence of the Simpson–Visser parameter on the thermodynamic behaviour can be understood directly from Fig. 1. Although the qualitative structure of the equation of state remains

identical to that of charged AdS black holes, the regularization parameter modifies the location of the critical point through the deformation of the effective horizon geometry. Increasing the parameter a suppresses the attractive interaction generated by the gravitational field and consequently shifts both the critical temperature and the critical pressure toward smaller values. As a result, the coexistence region gradually shrinks as the regularization becomes stronger.

The oscillatory part of the subcritical isotherms corresponds to a thermodynamically unstable branch characterized by a positive slope, $(\partial P/\partial V)_T > 0$, which violates the mechanical stability condition. In the physical description, this unstable segment is replaced by an isobar determined through Maxwell's equal-area construction, leading to a first order transition between small and large black hole configurations [7, 11]. The existence of this behaviour confirms that the charged Simpson–Visser–AdS black hole belongs to the same universality class as ordinary Van der Waals fluids despite the modifications introduced by the regular black hole geometry.

Another remarkable feature is that the regularization parameter does not destroy the critical phenomenon but only changes its quantitative characteristics. Therefore, the parameter a acts as a deformation parameter controlling the location of the critical point while preserving the overall thermodynamic structure. This observation will become even more evident in the Gibbs free energy analysis presented in the next subsection, where the first order phase transition is manifested through the characteristic swallowtail behaviour.

3.2 Gibbs Free Energy and First Order Phase Transition

The Gibbs free energy is the appropriate thermodynamic potential for black holes in the canonical ensemble, where the pressure and electric charge are kept fixed. It determines the globally preferred thermodynamic configuration through the minimum free energy principle and provides a direct criterion for identifying first order phase transitions.

For the charged Simpson–Visser–AdS black hole, the Gibbs free energy is defined as

$$G = M - TS, \quad (23)$$

where the mass M , Hawking temperature T , and entropy S are given by Eqs. (3), (6), and (12), respectively. Substituting these quantities into Eq. (23) yields the complete thermodynamic free energy as a function of the horizon radius, pressure, electric charge, and regularization parameter,

$$G = G(r_+, P, Q, a). \quad (24)$$

Unlike the equation of state, which characterizes local thermodynamic behaviour, the Gibbs free energy determines the global stability of competing black hole phases. Therefore, it is the most useful quantity for identifying the thermodynamic phase structure and the occurrence of first order transitions.

For sufficiently high temperatures, the Gibbs free energy is represented by a single smooth branch corresponding to one thermodynamically stable black hole phase. As the temperature decreases toward the critical value, different branches of the free energy gradually approach one another. Below the critical temperature, these branches intersect and form the characteristic

swallowtail structure, which is universally recognized as the signature of a first order phase transition in AdS black hole thermodynamics [7, 8].

The coexistence point is determined by the equality of Gibbs free energies,

$$G_{\text{Small}} = G_{\text{Large}}, \quad (25)$$

which corresponds to the transition between the small and large black hole phases. At this point the system minimizes its thermodynamic free energy by discontinuously changing its horizon radius while preserving the temperature and pressure.

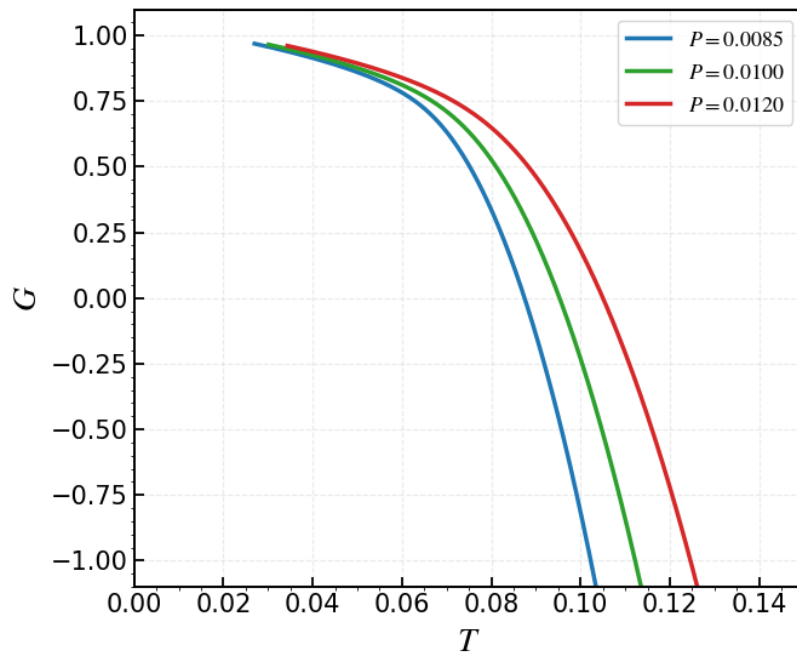


Figure 2: Gibbs free energy as a function of temperature for several values of the thermodynamic pressure. The appearance of the characteristic swallowtail structure below the critical pressure signals a first order phase transition between the small and large black hole phases. The red point denotes the critical point where the swallowtail disappears continuously.

Figure 2 clearly demonstrates the thermodynamic phase structure of the charged Simpson–Visser–AdS black hole. For pressures larger than the critical pressure, the Gibbs free energy consists of a single smooth branch, indicating the existence of only one thermodynamically stable phase. In this regime no phase transition occurs, and the system evolves continuously as the temperature increases.

A qualitatively different behaviour emerges when the pressure is reduced below the critical value. The Gibbs free energy develops the well known swallowtail structure, which represents the coexistence of multiple black hole solutions at the same temperature. Among these branches, the physically realized configuration is determined by the minimum Gibbs free energy. Consequently, the intersection point of the swallowtail corresponds to the first order transition between the small and large black hole phases.

The lower branch of the Gibbs free energy diagram represents the globally stable configuration, whereas the upper branch is thermodynamically disfavoured and therefore cannot represent an equilibrium state. As the temperature increases through the coexistence point, the preferred state changes discontinuously from the small black hole branch to the large black hole branch. This discontinuity is analogous to the liquid gas transition in an ordinary Van der Waals fluid and confirms the validity of the extended phase space interpretation.

The influence of the Simpson–Visser regularization parameter is primarily quantitative rather than qualitative. Increasing the parameter a shifts the location of the swallowtail and consequently modifies the coexistence temperature and the critical pressure. Nevertheless, the characteristic topology of the Gibbs free energy diagram remains unchanged, indicating that the regularization parameter preserves the universal mechanism responsible for the first order transition. This result suggests that the regular geometry alters the location of the thermodynamic critical point without changing the fundamental nature of the underlying phase transition.

From the holographic perspective, the swallowtail behaviour has an important interpretation. According to the AdS/CFT correspondence, the Gibbs free energy of the bulk black hole is identified with the free energy of the dual thermal conformal field theory living on the AdS boundary. Therefore, the discontinuous transition observed in Fig. 2 corresponds to a discontinuous change in the dominant thermal state of the boundary theory. In this sense, the regularization parameter a not only modifies the bulk thermodynamics but also influences the thermodynamic behaviour of the dual holographic system through its effect on the phase structure.

3.3 Heat Capacity and Thermodynamic Stability

The local thermodynamic stability of the charged Simpson–Visser–AdS black hole can be investigated through the heat capacity at constant pressure. According to the extended phase space formalism, the heat capacity is defined as

$$C_P = T \left(\frac{\partial S}{\partial T} \right)_P. \quad (26)$$

Using Eqs. (10) and (12), the heat capacity can be expressed as

$$C_P = 4\pi r_+ \frac{\frac{1}{\sqrt{r_+^2 + a^2}} - \frac{Q^2}{(r_+^2 + a^2)^{3/2}} + \frac{3\sqrt{r_+^2 + a^2}}{\ell^2}}{\frac{a^2 - r_+^2}{(r_+^2 + a^2)^2} + \frac{3Q^2 r_+^2}{(r_+^2 + a^2)^4} + \frac{3}{\ell^2}}. \quad (27)$$

The sign of C_P determines the local thermodynamic stability of the black hole configuration. A positive heat capacity corresponds to thermodynamically stable states, whereas negative values indicate unstable configurations. Divergences of the heat capacity occur when the denominator of Eq. (27) vanishes, signaling second order phase transitions in the canonical ensemble. A similar behavior has been observed through thermodynamic geometry in several modified gravity models [9, 12].

Figure 3 illustrates the variation of the heat capacity as a function of the horizon radius for several values of the regularization parameter a . The existence of vertical asymptotes separates the

parameter space into stable and unstable branches. Compared with the Reissner–Nordström–AdS case, the Simpson–Visser parameter shifts the locations of these divergences and consequently modifies the stability regions.

For small values of the horizon radius, increasing a suppresses the magnitude of the heat capacity divergence and moves the critical radius toward larger values. Physically, this behavior originates from the regularization of the central geometry, which weakens the singular behavior of the thermodynamic response. Consequently, the transition between the unstable small black hole branch and the stable large black hole branch occurs at larger horizon radii.

The large black hole branch remains thermodynamically stable for all considered values of the regularization parameter. In this regime, the heat capacity is positive and increases smoothly with the horizon radius, indicating that sufficiently large charged Simpson–Visser–AdS black holes are thermodynamically preferred. These results demonstrate that although the parameter a quantitatively modifies the critical points, it does not alter the qualitative stability structure of the system.

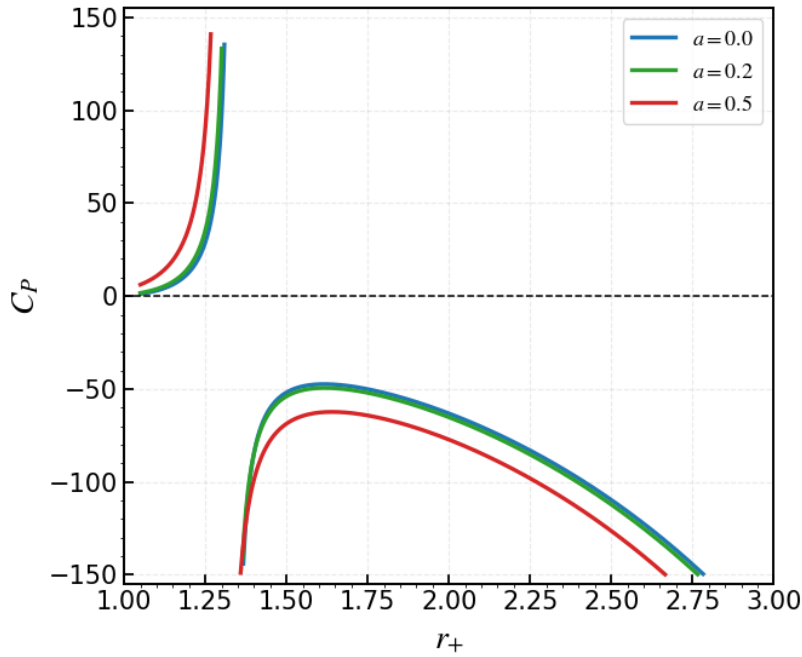


Figure 3: Heat capacity at constant pressure as a function of the horizon radius for several values of the Simpson–Visser parameter a . The divergences correspond to second order phase transitions separating unstable ($C_P < 0$) and stable ($C_P > 0$) thermodynamic branches.

3.4 Conclusions

The thermodynamic analysis presented in the previous subsections reveals that the charged Simpson–Visser–AdS black hole preserves the general characteristics of AdS black hole chemistry while exhibiting several important modifications induced by the regularization parameter a .

First, the presence of the Simpson–Visser parameter alters the entropy from the standard Bekenstein–Hawking area law. Consequently, all thermodynamic quantities inherit explicit corrections associated with the regular geometry. These modifications become increasingly significant for small

horizon radii, whereas the standard Reissner–Nordström–AdS behaviour is gradually recovered in the limit $a \rightarrow 0$.

Second, the equation of state exhibits the characteristic Van der Waals behaviour of charged AdS black holes. The corresponding critical point is shifted by the regularization parameter, leading to modified critical temperature, pressure, and thermodynamic volume. As a consequence, the universal ratio

$$\frac{P_c V_c}{T_c} = \frac{3}{8} \left(\frac{6Q^2}{6Q^2 + a^2} \right)^{3/2}, \quad (28)$$

deviates from the universal Reissner–Nordström–AdS value. This result demonstrates that the Simpson–Visser regularization introduces quantitative corrections to the thermodynamic universality class while preserving the overall phase structure.

The Gibbs free energy analysis further confirms the existence of a first order small/large black hole phase transition below the critical temperature. The characteristic swallowtail structure persists for nonzero values of the regularization parameter, although its location and size depend explicitly on a . Therefore, the regular geometry influences the coexistence region without changing the qualitative nature of the phase transition.

Finally, the heat capacity analysis establishes the local thermodynamic stability of the system. The divergence points of C_P separate stable and unstable thermodynamic branches and correspond to second order phase transitions. Increasing the regularization parameter shifts these divergence points toward larger horizon radii, enlarging the parameter region occupied by thermodynamically stable large black holes.

Overall, the charged Simpson–Visser–AdS black hole provides a consistent example of a regular AdS spacetime whose thermodynamic behaviour closely resembles that of conventional charged AdS black holes while exhibiting measurable corrections generated by the regularization parameter. These corrections are reflected in the critical quantities, free energy landscape, and local stability properties, making this geometry an interesting framework for further investigations in black hole chemistry and holographic thermodynamics.

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Full paper - Presentation

Thermal string field theory, Hagedorn scaling, and de Sitter observer algebras

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Abstract. Quantum gravity in de Sitter space lacks the asymptotic observables available in flat or anti-de Sitter backgrounds, and the natural observables are instead organized into the operator algebra of a static-patch observer, which is known to be a finite von Neumann algebra of type II_1 . A conditional route from finite-temperature closed string field theory to this observer algebra is developed here. Compact Euclidean time produces, for closed strings, thermal winding sectors in addition to the usual Matsubara modes, and the Atick–Witten Hagedorn transition obstructs a naive high-temperature limit. The regime beyond the transition is formulated as an effective scaling algebra with a nonzero thermal winding condensate, and a leading truncation is exhibited whose radial collective mode supplies explicit low-point observables. It is proposed that this collective sector may be captured by a double-scaled SYK description, in the precise sense of matching renormalized correlators, spectral moments and operator growth data after a fixed normalization and an affine time reparametrization. The proposed operator algebraic completion conditionally separates a microscopic type III_1 horizon algebra from the finite type II_1 observer algebra, related by the modular crossed product and a finite corner. Only the proposed results are stated in this paper, and the complete construction with the corresponding derivations is being developed in the companion work.

Keywords: String Field Theory; de Sitter Space; Holography; Operator Algebras; Double-Scaled SYK.

1 Introduction

A quantum theory of de Sitter space cannot be formulated in terms of asymptotic observables in the way familiar from flat-space or anti-de Sitter quantum gravity. The relevant observables are tied to an observer and to the associated cosmological horizon, and the natural replacement is an algebraic description of the static patch, in which the observer algebra is a finite von Neumann algebra of type II_1 [1]. Local quantum field theory and modular theory naturally produce type III von Neumann algebras [2], so the finite observer algebra must emerge from a type III microscopic description in some indirect way. Crossed product constructions in gravitational systems [3] indicate the general mechanism which relates the two structures, and the question addressed here is which microscopic theory supplies the type III algebra in the de Sitter case.

A candidate microscopic model is provided by the double-scaled SYK system, in which one starts from N Majorana fermions with p -body random interactions and takes the limit

$$N \rightarrow \infty, \quad p \rightarrow \infty, \quad p^2/N = \lambda \text{ fixed}, \quad (1)$$

where correlation functions are organized by chord diagrams. The infinite-temperature sector of this model has been proposed to capture essential aspects of de Sitter holography [4, 5, 6], and its operator algebra is a finite tracial algebra of type II_1 , which is precisely the algebraic character expected of static-patch observables. This picture is taken here as the holographic input, and the question becomes whether a sector with the same infinite-temperature character can be motivated from string theory itself. The framework used for this purpose is finite-temperature closed string field theory [7, 8], and the obstruction to a naive high-temperature limit, namely the Hagedorn transition [9], becomes the constructive tool. The phase beyond the transition is modeled as a scaling algebra, compared with the double-scaled SYK sector, and placed in a proposed algebraic chain toward the finite observer algebra of de Sitter space. Only the proposed results are summarized in what follows, and the complete construction together with all derivations is being developed in the companion work [16].

2 Thermal winding and the Hagedorn obstruction

At inverse temperature β , Euclidean target-space time is compactified as $X_E^0 \sim X_E^0 + \beta$. For point particles this produces the standard Matsubara modes, but a closed string can also wind around the Euclidean thermal circle according to

$$X_E^0(\sigma + 2\pi, \tau) = X_E^0(\sigma, \tau) + w\beta, \quad w \in \mathbb{Z}, \quad (2)$$

so finite-temperature closed string theory is graded by both thermal momentum and thermal winding [9, 10]. In the bosonic normalization $\alpha' = 1/2$, the lowest thermal winding mode has the mass

$$M^2(\beta) = -8 + \beta^2/\pi^2, \quad \beta_H = 2\pi\sqrt{2}, \quad (3)$$

and it becomes massless at the Hagedorn point and tachyonic for $\beta < \beta_H$. The same obstruction appears in the asymptotic density of string states, which grows as

$$\rho(E) \sim CE^{-a} \exp(\beta_H E), \quad (4)$$

and this growth makes the canonical partition function diverge for $\beta < \beta_H$. This behavior is interpreted as a genuine phase transition [9], with a high-temperature phase whose free energy grows far more softly than in any local relativistic field theory, and the soft growth signals a drastic reduction of the effective degrees of freedom. The route toward $\beta \rightarrow 0$ therefore crosses a stringy phase governed by thermal winding, and the high-temperature sector cannot be obtained by retaining only the Matsubara momentum modes.

3 The post-Hagedorn scaling sector

Within the leading truncation used here, the string field is expanded around a putative configuration of the form $\Psi = \Psi_{\text{Hag}} + \chi$, where Ψ_{Hag} is associated with condensation of the lowest thermal winding mode and χ is the fluctuation. When only the lowest winding states are retained, the thermal scalar ϕ acquires an effective potential with a quartic stabilization, and its nontrivial stationary point below β_H is the winding condensate [9, 11] given by

$$|\phi_{\text{Hag}}|^2 = (\beta_H^2 - \beta^2)/(\pi^2 \lambda_H), \quad (5)$$

where λ_H is the effective quartic coupling of the winding sector. The radial fluctuation of the condensate, canonically normalized as $q(s)$, is governed by a single residual collective time parameter s inherited from the scaling limit, and its Gaussian two-point function takes the form

$$G^{(2)}(s) = \exp(-\mu_H |s|)/(2\mu_H), \quad \mu_H^2 = 2(\beta_H^2 - \beta^2)/\pi^2, \quad (6)$$

while the connected four-point function begins at interaction order. These expressions give the first explicit observables of the effective high-temperature phase, and the physical picture behind them is that a single collective coordinate survives out of the full thermal string spectrum. The high-temperature sector is defined algebraically, within this proposal, by considering families of admissible finite-temperature operators whose renormalized correlation functions possess a finite limit after the background shift and the high-temperature scaling, and by removing the null ideal $\mathcal{N}_{\text{Hag}}$ of families whose renormalized matrix elements vanish in the limit, which produces the effective algebra

$$\mathcal{A}_{\text{TST}}^{\text{Hag},\infty} = \mathcal{A}_{\text{Hag}}/\mathcal{N}_{\text{Hag}}. \quad (7)$$

The quotient implements in operator algebraic language the proposed reduction of degrees of freedom described above, because massive modes, gauge and BRST-exact directions and non-surviving fluctuations are removed, and the remaining sector is generated by the collective winding mode $O_{\text{wind}}(s)$ together with its residual evolution. Only this collective sector is effectively one-dimensional, and the target space itself is not reduced.

4 Comparison with the double-scaled SYK infinite-temperature sector

In the double-scaled limit (1), disorder-averaged correlators are organized by chord diagrams, and the infinite-temperature state is the trace τ_∞ on the chord algebra [12, 13]. The moments of the chord Hamiltonian are counted by pairings weighted by the crossing parameter q , and the first nontrivial values are

$$\tau_\infty(H_{\text{ch}}^2) = 1, \quad \tau_\infty(H_{\text{ch}}^4) = 2 + q, \quad \tau_\infty(H_{\text{ch}}^6) = 5 + 6q + 3q^2 + q^3. \quad (8)$$

The comparison proposed here is a state-preserving map Φ from the collective Hagedorn sector to the chord sector, which identifies the winding collective operator with a chord operator after a fixed normalization and an affine reparametrization of time according to

$$\Phi(O_{\text{wind}}(s)) = Z_O^{1/2} C(as + b), \quad \Phi(H_{\text{Hag}}) = a^{-1} H_{\text{ch}} + c \mathbf{1}. \quad (9)$$

The comparison is overconstrained by design. The same constants (a, b, c, Z_O) fixed by the two-point function must then reproduce, without further freedom, the connected four-point function, the spectral moments of the residual Hamiltonian and the operator growth function obtained by mapping the winding size operator to the chord number operator, so a failure at any of these levels would falsify the proposal. Recent de Sitter oriented developments of the double-scaled SYK model [6, 14] supply the observer side comparison data, and the element added here is the proposed post-Hagedorn string theoretic origin of that data.

5 Operator algebras and the de Sitter interpretation

Assuming the post-Hagedorn scaling state is faithful on the horizon subalgebra, its GNS representation defines a von Neumann algebra M_H with a cyclic and separating vector, so Tomita–Takesaki theory applies. The residual Hagedorn dynamics is assumed to agree with the modular

flow up to a normalization. It is proposed that the horizon algebra is a factor of type III_1 ; this classification is not derived here, and a derivation of the Connes invariant $S(M_H) = [0, \infty)$ from the scaling state is left for future work. Under these assumptions, the finite observer algebra arises through the standard chain [1, 3] given by

$$N_H = M_H \rtimes_{\sigma} \mathbb{R} \quad (\text{type II}_{\infty}), \quad M_{\text{obs}} = e N_H e \quad (\text{type II}_1), \quad (10)$$

where the crossed product by the modular flow is the semifinite continuous core with trace Tr_H and e is a projection of finite trace. The entropy scale is matched by imposing the trace normalization

$$\log \text{Tr}_H(e) = S_{\text{dS}} = \text{Area}(H)/4G_N, \quad (11)$$

which matches the Gibbons–Hawking entropy [15], while the normalized trace on the corner gives the infinite-temperature observer state. In the proposed correspondence, the finite tracial double-scaled SYK algebra is embedded trace-preservingly in M_{obs} rather than identified with the microscopic horizon algebra. The infinite SYK temperature and finite Gibbons–Hawking temperature $T_{\text{dS}} = 1/(2\pi R)$ play different roles: the former refers to the tracial observer-side state, while the latter enters through the modular normalization of physical static-patch time and the choice of thermal observer state within the finite algebra.

6 Conclusions

Finite-temperature closed string field theory differs from ordinary thermal field theory because the Euclidean thermal circle supports winding, and the resulting Hagedorn transition forbids a naive high-temperature limit. The regime beyond the transition was modeled as a post-Hagedorn scaling algebra with a nonzero winding condensate, its leading collective observables were exhibited, and a testable overconstrained matching of this effectively one-dimensional sector with the infinite-temperature chord sector of the double-scaled SYK model was proposed. The proposed operator algebraic completion conditionally separates a type III_1 horizon algebra from the finite type II_1 de Sitter observer algebra, related by the modular crossed product and a finite corner whose trace normalization matches the de Sitter entropy. The remaining dynamical input is concentrated in three tasks left for future work: completion of the leading condensate to a full shifted closed string field theory solution including massive modes and backreaction, derivation of the type III_1 property from the modular spectrum of the scaling representation, and extension to a stable superstring setting. The corresponding calculations and complete derivations are being developed in the companion paper [16].

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Abstract - Presentation

Pion Charge Density in the Thermal Soft-Wall Model of Holographic QCD

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Abstract. We investigate the pion transverse charge density, $\rho_\pi(b, T)$, at finite temperature within the framework of the thermal soft-wall model of holographic QCD. The pion electromagnetic form factor is extended from its zero-temperature counterpart by incorporating thermal effects into the holographic description, allowing us to study the modification of the pion internal structure in a hot medium. We find that both the pion charge density and the electromagnetic form factor decrease monotonically with increasing temperature due to thermal fluctuations. As the temperature approaches the critical temperature, T_c , these quantities gradually vanish, indicating the disappearance of the pion as a well-defined bound state and signaling the onset of deconfinement and chiral symmetry restoration. The obtained thermal behavior is consistent with the expected properties of low-energy QCD and demonstrates that the thermal soft-wall model provides a reliable phenomenological framework for describing the temperature evolution of pion observables under extreme conditions.

Keywords: Thermal soft-wall model, holographic QCD, pion electromagnetic form factor, transverse charge density, finite temperature, chiral symmetry restoration, deconfinement transition

Abstract - Presentation

Machine Learning in Holographic QCD Phase Transitions

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Abstract.

Machine learning offers a data-driven approach to refining holographic QCD models and investigating the phase structure of strongly interacting matter. In this talk, we present three complementary applications: parameter optimization, uncertainty quantification, and inverse problems. First, we employ gradient-based optimization to calibrate an analytical Einstein–Maxwell–dilaton model against thermodynamic data, including the equation of state, baryon number susceptibility, and baryon number density. Second, we discuss Bayesian inference as a framework for constraining model parameters and quantifying uncertainties in the predicted QCD critical end point. Third, we introduce neural networks to represent the warp factor, allowing the holographic background to be reconstructed from data rather than restricted to a prescribed analytical parametrization. This strategy is illustrated through an Einstein–dilaton model for pure Yang–Mills thermodynamics and the scalar glueball spectrum. We also discuss related applications to heavy quarkonium and neural-network holographic models constrained by lattice QCD and hadron resonance gas inputs. These examples highlight how machine learning can complement holographic modeling through improved parameter determination, uncertainty assessment, and data-driven reconstruction of the bulk geometry.

Keywords: Holographic QCD; Machine Learning; QCD Phase Transition; Bayesian Inference; Inverse Problems.

Abstract - Presentation

Probing Chiral and $U(1)$ Axial Symmetry Restoration via Meson Susceptibilities in Holographic QCD

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Abstract. We investigate the restoration patterns of chiral and $U(1)$ axial symmetries at finite temperature within a soft-wall holographic QCD framework. Utilizing two distinct parameter sets calibrated to the physical pion mass and a pseudocritical temperature $T_{pc} \sim 155$ MeV, we analyze the temperature dependence of the light and strange quark condensates. The results confirm a smooth chiral crossover transition, yielding pseudocritical temperatures of 157 MeV and 154 MeV for the respective cases. Near T_{pc} , the screening masses of the chiral partner mesons (π - σ and η - a_0) become degenerate, providing a clear signature of chiral symmetry restoration, which is further supported by our analysis of the corresponding meson susceptibilities. Conversely, the effective restoration of the $U(1)_A$ symmetry is found to occur more slowly than that of the chiral symmetry. This behavior indicates a clear separation between the chiral and axial symmetry restoration scales within this holographic framework. Additionally, we compute the temperature-dependent topological susceptibility $\chi_{top}^{1/4}$, which exhibits a sharp drop near T_{pc} . While our model qualitatively captures the established features of the chiral transition, the results highlight current quantitative limitations in describing the $U(1)$ axial anomaly compared to recent Lattice QCD data.

Keywords: Holographic QCD; Chiral Symmetry; $U(1)$ axial symmetry ; Meson susceptibility; Topological susceptibility.

Abstract - Presentation

Dynamical Gravitational Entropy Is a Noether Charge

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Abstract. Black hole thermodynamics in generic dynamical, non-equilibrium regimes remains a fundamental challenge. We establish dynamical entropy as a Noether charge associated with a generic evolving null surface subject to Dirichlet boundary conditions. We identify the symmetry generator underlying this dynamical entropy as a null vector on the null surface, determined by physically motivated geometric conditions that lead to a notion of a “dynamical zeroth law.” We further show that the corresponding Noether charge density satisfies the second law locally and at each instant in time, without invoking the teleological final conditions characteristic of event horizons. Our construction applies to generic dynamical gravitational systems without assuming background stationarity, providing a local formulation of dynamical entropy and its associated second law.

Keywords: Dynamical Gravitational Entropy; Noether Charge; Second Law; Black Holes.

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